Linear Algebra Course

**Overview**

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| Level | 1 (Semester 2) |
| Duration | 4 weeks |
| Lectures | 10 times 40-minute lectures per week for 3 weeks |
| Practicals/tutorials | 5 times 40 min tutorial per week for 3 weeks |

# Course Aims

There are three main aims of the course. These are (1) to give an introduction to basic mathematical and algebraic concepts like sets, relations, functions, groups, rings, fields, (2) to give a solid introduction to commonly used methods and results from linear algebra, and (3) to give the students an impression of how a mathematical theory is build up through definitions, lemmas/propositions/theorems and proofs.

# Learning Objectives

By the completion of the course a student should be able to

* work with sets, relations, functions from the viewpoint of naive set theory;
* define the basic concepts of Linear algebra, such as a vector space over a field, a linear transformation etc., and understand the connections among linear transformations, matrices and systems of linear equations;
* define linear independence, span and related notions and learn how to determine independence;
* prove simple results about matrices;
* systematically solve a system of linear equations;
* describe row reduction in terms of elementary matrices, and prove simple results in terms of elementary matrices;
* find the inverse of a given matrix, if it exists, and know how to determine whether or not it does;
* define vector subspaces of a vector space, know how to determine if a given subset is a vector subspace, and apply to certain vector subspaces associated with a matrix and/or a linear transformation;
* prove elementary results about vector spaces and the connection with matrices and systems of linear equations;
* define a basis of a finite dimensional vector space, find a basis for a given vector space, define dimension, and know how to calculate it;
* prove elementary results about bases and dimension;
* define rank and nullity and know how to calculate them;
* define and calculate the determinant of a matrix;

# Syllabus

* **Basic set theory:** Naive understanding of a set and of relations, in particular equivalence relations. Maps (injective/surjective/bijective). The principal of induction.
* **Groups, Rings, Fields. Solving a linear system over a field.** Definition and examples of groups, rings and fields, in particular the field Fp. Elementary row operations, row echelon form, Gaussian algorithm for solving a linear system over a field.
* **Vector spaces and K-algebras.** Definition of a vector space over a field. Examples. Subspaces of a vector space, intersection and sum of subspaces. Span, spanning sets. Linear independence. Basis, dimension. Elementary results about bases and dimension. Change of basis matrix. Definition of a (commutative) K-algebra. The algebra of nxn-matrices.
* **Linear maps.** Definition of a linear map between two K-vector spaces. Kernel, image, injective, surjective linear maps. Matrix of a linear map. Rank of a matrix. Invertible

matrices. Determinants. Change of basis and the matrix of a linear map. The algebra of endomorphisms and the K-algebra homomorphism between the algebra of nxn-matrices and the endomorphism algebra of a vector space of dimension n.

* **Determinants of matrices and linear transformations.** Will be treated in more detail in MA2508.