

# Plasmon Excitation on a Metal Grating and its Phase Detection Based Biosensor Application

Ziqian Luo<sup>1</sup>, Taikei Suyama<sup>2</sup>, Yoichi OKuno<sup>2</sup>

1. Centre for Optical and Electromagnetic Research, South China Normal University, Guangzhou, China

2. Graduate School of Science and Technology, Kumamoto University, Kumamoto, Japan

e-mail address: einshim@gmail.com

**Abstract:** A grating-based biosensor via phase detection is stated. The resolution will be  $10^{-7}$  RIU in the state-of-the-art today. The workspace can be shifted in a large range while the sensor resolution can remain in a high level by only changing the incident angle.

**OCIS codes:** (050.0050) Diffraction and gratings; (050.5080) Phase shift

## 1. Introduction

An interesting property of a metal grating is known as the resonance absorption: partial or total absorption of incident light energy occurs at a specific angle of incidence (resonance angle). This is caused by excitation of plasmon surface waves and is accompanied by an abrupt change of diffraction efficiency [1].

We know that the resonance absorption occurs when a phase-matching condition is satisfied: the phase constant of an evanescent order coincides with the real part of an eigenvalue of surface plasmons. According to our previous research, we can easily determine the refractive index of the material with an accuracy of five digits by measuring the zeroth-order efficiency alone [2]. The five-digit resolution, however, is not sufficient for practical applications [3,4]. Hence, we have an urgent issue to increase the resolution in the index determination.

A promising solution to our issue is to employ phase detection to measure the phase modulation accompanying the plasmon excitation. This is because the Fresnel models of surface plasmon excitation predict an obvious change in the reflected phase as the material index changed [5]. The phase detection has already been employed in prism-based biosensors and works to improve the resolution of the sensors to  $10^{-7}$  refractive index units (RIU) [6].

In the following sections we examine the possibility of the phase detection employed in the grating-based plasmon biosensor. We will formulate the problem of diffraction by a metal grating placed in conical mounting and describe the method of solution employed. We will also show the numerical results and will make discussions from the viewpoint of sensor applications.

## 2. Formulation of the Problem and the Method of Solution

Figure 1 shows the schematic representation of diffraction by a layered grating made of a metal and having an over-coating made of another metal. The grating is uniform in the  $Y$  direction and is periodic in  $X$ . The surface profiles are given by

$$S_1: z_1 = \eta_1(x) = h \sin(2\pi x/d) \quad (1a)$$

$$S_2: z_2 = \eta_2(x) = h \sin(2\pi x/d) - e \quad (1b)$$

where  $h$ ,  $d$ , and  $e$  are the amplitude (half depth) of the surface modulation, the period, and the thickness of the coating. Note that the small letters ( $x$ ,  $y$ ,  $z$ ) denote a point on the surfaces. The capital letters ( $X$ ,  $Y$ ,  $Z$ ) show the coordinates of a point in these regions. A convention  $\mathbf{P} = (X, Y, Z)$  will be used as well.

The electric field of an incident light is given by

$$\mathbf{E}^i(\mathbf{P}) = \mathbf{e}^i \exp(i\mathbf{k}^i \cdot \mathbf{P} - i\omega t) \quad (2)$$

where

$$\mathbf{k}^i = (\alpha, \beta, -\gamma) \quad (3)$$

with  $\alpha = n_1 k^i \sin \theta \cos \phi$ ,  $\beta = n_1 k^i \sin \theta \sin \phi$ ,  $\gamma = n_1 k^i \cos \theta$ , and  $k^i = 2\pi/\lambda$ . As shown in Fig.1,  $\theta$  is the polar angle between the  $Z$ -axis and the incident wave-vector,  $\phi$  is the azimuth angle between the  $X$ -axis and the plane of incidence, and  $\lambda$  is the wavelength in vacuum.

Let us decompose the amplitude  $\mathbf{e}^i$  appeared in (2) into a TE- and a TM-component, where TE (or TM) means the absence of the  $Z$ -component in the relevant electric (or magnetic) field. Apparently,  $\mathbf{e}^{\text{TE}}$  has no  $Z$ -component. The amplitude  $\mathbf{e}^i$  is decomposed as

$$\mathbf{e}^i = \mathbf{e}^{\text{TE}} \cos \delta + \mathbf{e}^{\text{TM}} \sin \delta \quad (4)$$

where  $\delta$ , which is termed a polarization angle, is the angle between  $\mathbf{e}^{\text{TE}}$  and  $\mathbf{e}^i$  (see Fig.1).

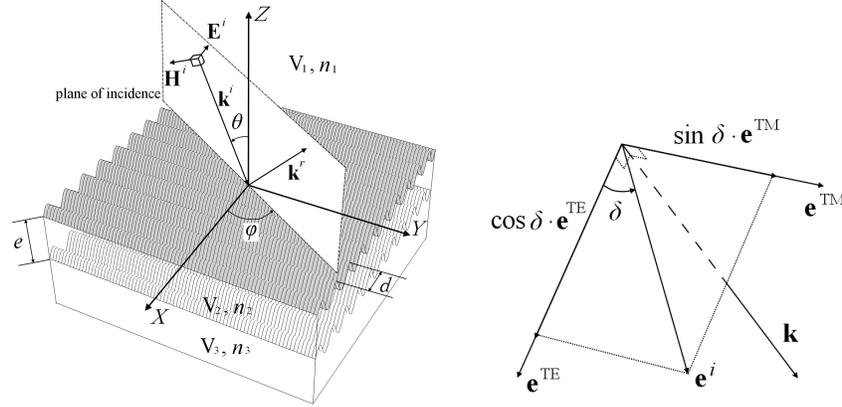


Fig.1. Schematic representation of diffraction by a layered metal grating and definition of the polarization angle.

According to Yasuura's theory [7, 8], The approximate solutions in each region, hence, can be expressed as:

$$\begin{aligned}
 \begin{pmatrix} \mathbf{E}_{1N}^d \\ \mathbf{H}_{1N}^d \end{pmatrix}(\mathbf{P}) &= \sum_{n=-N}^N A_{1n}^{\text{TE}+}(N) \begin{pmatrix} \phi_{1n}^{\text{TE}+} \\ \varphi_{1n}^{\text{TE}+} \end{pmatrix}(\mathbf{P}) + \sum_{n=-N}^N A_{1n}^{\text{TM}+}(N) \begin{pmatrix} \phi_{1n}^{\text{TM}+} \\ \varphi_{1n}^{\text{TM}+} \end{pmatrix}(\mathbf{P}) \\
 \begin{pmatrix} \mathbf{E}_{2N}^d \\ \mathbf{H}_{2N}^d \end{pmatrix}(\mathbf{P}) &= \sum_{n=-N}^N A_{2n}^{\text{TE}-}(N) \begin{pmatrix} \phi_{2n}^{\text{TE}-} \\ \varphi_{2n}^{\text{TE}-} \end{pmatrix}(\mathbf{P}) + \sum_{n=-N}^N A_{2n}^{\text{TM}-}(N) \begin{pmatrix} \phi_{2n}^{\text{TM}-} \\ \varphi_{2n}^{\text{TM}-} \end{pmatrix}(\mathbf{P}) \\
 &\quad + \sum_{n=-N}^N A_{2n}^{\text{TE}+}(N) \begin{pmatrix} \phi_{2n}^{\text{TE}+} \\ \varphi_{2n}^{\text{TE}+} \end{pmatrix}(\mathbf{P}) + \sum_{n=-N}^N A_{2n}^{\text{TM}+}(N) \begin{pmatrix} \phi_{2n}^{\text{TM}+} \\ \varphi_{2n}^{\text{TM}+} \end{pmatrix}(\mathbf{P}) \\
 \begin{pmatrix} \mathbf{E}_{3N}^d \\ \mathbf{H}_{3N}^d \end{pmatrix}(\mathbf{P}) &= \sum_{n=-N}^N A_{3n}^{\text{TE}-}(N) \begin{pmatrix} \phi_{3n}^{\text{TE}-} \\ \varphi_{3n}^{\text{TE}-} \end{pmatrix}(\mathbf{P}) + \sum_{n=-N}^N A_{3n}^{\text{TM}-}(N) \begin{pmatrix} \phi_{3n}^{\text{TM}-} \\ \varphi_{3n}^{\text{TM}-} \end{pmatrix}(\mathbf{P})
 \end{aligned} \tag{5}$$

where  $N$  is the number of truncation. We determine the coefficients by the least-squares method. That is, we find the coefficients that minimize the quadratic form:

$$\begin{aligned}
 I_N &= \int_{S_1} |\nu \times [E_{1N}^d + E^i - E_{2N}^d] \cdot s|^2 ds + \int_{S_2} |\nu \times [E_{2N}^d - E_{3N}^d] \cdot s|^2 ds \\
 &\quad + W^2 \int_{S_1} |\nu \times [H_{1N}^d + H^i - H_{2N}^d] \cdot s|^2 ds + W^2 \int_{S_2} |\nu \times [H_{2N}^d - H_{3N}^d] \cdot s|^2 ds
 \end{aligned} \tag{6}$$

where  $S_1$  and  $S_2$  denote one period of the upper and the lower surface,  $\nu$  is a unit normal vector to the surfaces, and  $W$  is an intrinsic impedance of vacuum. This over-determined set of equations is solved approximately by the QR-decomposition.

### 3. Numerical Results and Discussion

In the following, we assume the use of a commercial grating made of aluminum with an gold over-coating, whose parameters are  $2h = 0.072 \mu\text{m}$ ,  $d = 0.556 \mu\text{m}$  and  $e = 0.778 \mu\text{m}$ . The incident light is a monochromatic plane wave from a laser diode with a wavelength of  $0.633 \mu\text{m}$ . As for the value taken in our computation, we assume that the refractive indices of Al and Au at this wavelength are  $n_2 = 0.1594 + i3.2166$  and  $n_3 = 1.2078 + i7.0148$ .

The phase signal of the zeroth-order TM diffracted mode in  $V_1$  is defined as phase shift between before and after SPR activated. The phase signal of the zeroth-order TM mode is then given by

$$\varphi_{\text{phase}} = \sin^{-1} \left[ \frac{\text{Im}(A_0^{\text{TM}})}{\sqrt{\text{Re}(A_0^{\text{TM}})^2 + \text{Im}(A_0^{\text{TM}})^2}} \right] \tag{7}$$

Figure 2a shows the efficiency of  $\text{TM}_0$  mode as a function of the incident polar angle  $\theta$  for different azimuth angles while  $V_1$  is filled with a dielectric of refractive index  $n_1 = 1.330$ . Each curve in the figure shows a resonant incident polar angle  $\theta$  for a given azimuth angle  $\varphi$ . It is easily to be observed that the absorption efficiency of SPR is strongly related to the azimuth angle  $\varphi$ . Correspondingly, Fig. 2b shows the phase shift of  $\text{TM}_0$  mode while the refractive index  $n_1$  changes around 1.330. The phase shift curve has a steep slope over a limited range of refractive index. We also notice that the slopes of phase shift curves are related with the strength of the resonance absorption. That is, the strongest resonance with almost total absorption observed at  $\theta = 20.5^\circ$  and  $\varphi = 11.2^\circ$  is accompanied with the most abrupt slope of the phase shift curve.

Figure 2b shows great potential of a grating-based biosensor with phase detection for the purpose of high resolution. Obviously, the resolution of such a sensor design is related to the slope of a curve chosen from the curves in Fig. 2b. Although a steeper slope means higher resolution, the steepest curve in Fig. 2b ( $\theta = 20.5^\circ$  and  $\varphi = 11.2^\circ$ ) is difficult to be employed in practice because of an extremely low efficiency far less than  $10^{-3}$ . Therefore, we have a trade-off between high resolution and a sufficient diffracted power. For example, we can choose the secondary tilted current ( $\theta = 20.3^\circ$  and  $\varphi = 5.6^\circ$ ) in experiment to ensure that the signal beam is strong enough to be detected. If we choose these parameters, and if we refer to an experimental design whose instrumental resolution in phase detection is  $5 \times 10^{-3}$  degrees [9, 10], then we can expect the resolution in refractive index should be about  $10^{-7}$  RIU.

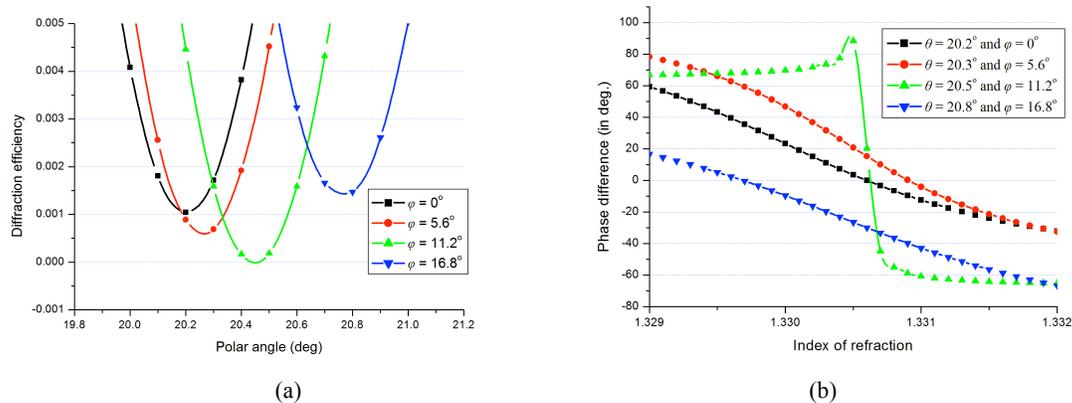


Fig. 2. (a) Diffraction efficiency as a function of polar angle for given azimuth angles. (b) Phase response for different incident angles.

We scan the resonance phenomenon around the index range as  $1.271 < n_1 < 1.347$ , and find a very similar result as Fig. 2. Thus, it is obviously that without changing the structure parameters but only the incident angle, we can shift the workspace in a large range while the sensor resolution can remain in a high level. Combining all the small workspaces, a wide range of refractive index can be determined respectively.

#### 4. Conclusions

Since the sample index  $n_1$  has large influence on the phase modulation of the diffracted TM mode, the phase detection is a strong way to find the index. The resolution of the possible grating-based biosensor with phase detection will be  $10^{-7}$  RIU in the state-of-the-art today. This means more than 7-digit determination of the index and can be employed for medical and physiological applications. Moreover, by only changing the incident angle, the workspace of such a biosensor can be shift in a larger range.

#### 5. Acknowledgment

This project was Supported by Guangdong Innovative Research Team Program(No. 201001D0104799318).

#### 6. References

- [1] H. Raeter, Surface plasmon and roughness, in V.M. Argranovich and D.L. Mills (eds.), *Surface Polaritons*, Chap. 9, 331–403, North-Holland, New York, 1982.
- [2] T. Suyama and Y. Okuno, Enhancement of TM-TE mode conversion caused by excitation of surface plasmons on a metal grating and its application for refractive index measurement, *Progress In Electromagnetics Research*, **72**, 91–103, 2007.
- [3] S.G. Nelson, K.S. Johnston, and S.S. Yee, High sensitivity surface plasmon resonance sensor based on phase detection, *Sensors and Actuators B*, **35-36**, 187–191, 1996.
- [4] Banerjee, A., Enhanced refractometric optical sensing by using one-dimensional ternary photonic crystals, *Progress In Electromagnetics Research*, **89**, 11–22, 2009.
- [5] H.P. Ho, W.C. Law, S.Y. Wu, Chinlon Lin, and S.K. Kong, Real-time optical biosensor based on differential phase measurement of surface plasmon resonance, *Biosensors and Bioelectronics*, **20**, 2177–2180, 2005.
- [6] H.P. Ho, W. Yuan, C.L. Wong, S.Y. Wu, Y.K. Suen, S.K. Kong, and Chinlon Lin, Sensitivity enhancement based on application of multi-pass interferometry in phase-sensitive surface plasmon resonance biosensor, *Optics Communications*, **275**, 491–496, 2007.
- [7] K. Yasuura and T. Itakura, Approximation method for wave functions (I), (II) and (III), *Kyushu Univ. Tech. Rep.*, **38**, 1, 72–77, 1965; **38**, 4, 378–385, 1966; **39**, 1, 51–56, 1966.
- [8] Y. Okuno and H. Ikuno, Yasuura's method, its relation to the fictitious source methods, and its advancements in solving 2-D problems, in Th. Wriedt (ed.), *Generalized Multipole Techniques for Electromagnetic and Light Scattering*, 111–141, Elsevier, Amsterdam, 1999.
- [9] Andrei V. Kabashin, Sergiy Patskovsky, Alexander N. Grigorenko, Phase and amplitude sensitivities in surface plasmon resonance bio and chemical sensing, *Optics Express*, **17**, 23, 21191–21204, 2009.
- [10] P. Horowitz and W. Hill, *The Art of Electronics*, 641–646, Cambridge University Press, Cambridge, 1989.