Sets and Axioms

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Outline

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What is a set

A set is a collection into a whole of definite, distinct objects of our intuition or our thought. The objects are called elements (members) of the set.

Georg Cantor

Russell's paradox

$$\{x|x \notin x\}$$

(First-order) Language for sets

- variables: $x, y, z, x_0, x_1, \ldots$.
- connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- quantifiers: $\forall, \exists$
- predicate symbols: $=, \in$
- auxiliary symbols:) , (.

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Formulas of the language $\mathcal{L}_S$

- If t_1 and t_2 are variables, then $(t_1 = t_2)$ is a formula.
- If α is a formula, so is $\neg\alpha$.
- If α and β are formulas, so is $(\alpha \star \beta)$, where $\star$ is $\wedge$, $\vee$, $\rightarrow$, or $\leftrightarrow$.
- If α is a formula, then for any variable x, $\forall x\alpha$ and $\exists x\alpha$ are also formulas.
- Formulas are exactly those expressions obtained by the above rules.

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Deductions in $\mathcal{L}_S$

$$\Sigma \vdash \alpha$$

ZFC

- ZFC $= \{\exists x \forall y (y \notin x), \ldots\}$
- A proof of α in ZFC = a deduction for ZFC $\vdash \alpha$
- We usually make statements and proofs **informally** in the axiomatic set theory!

Example (for axioms)

- **The Axiom of Existence**: There exists a set which has no elements.
- **The Axiom of Extensionality**: If two sets have the same elements, then they are identical.

- $\exists x \forall y (y \notin x)$
- ...

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- $\ldots$

Example (for proofs)

- Prove: There exists only one set with no elements.
- ZFC $\vdash \exists! x \forall y (y \notin x)$

Example (for definitions)

- Definition: The (unique) set with no elements is called the **empty set** and is denoted $\varnothing$.
- By the above definition, we introduce a new symbol into the language $\mathcal{L}_S$.
- New symbols serve as a simplification medium. If we like, we can always delete the new symbol.
- $x = \varnothing$: $\forall y(y \notin x)$
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What is a property

- A property of x is a formula of form $\alpha(x)$.
- A property of x and y is a formula of form $\alpha(x, y)$, that is, a formula with free variables among x and y

- $x = x$.
- $x = \varnothing$.
- $x \notin x$.
- $x = y$
- $x \notin y$ or $x \in z$ (formally, $x \notin y \vee x \in z$)

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The Axiom Schema of Comprehension

- Let $P(x)$ be a property of x. For any set A, there is a set B such that $x \in B$ if and only if $x \in A$ and $P(x)$.
- Formally, $\forall z \exists y \forall x (x \in y \leftrightarrow x \in z \wedge \alpha(x))$

- Lemma: For every A, there is only one set B such that $x \in B$ if and only if $x \in A$ and $P(x)$.

- Definition: $\{x \in A | P(x)\}$ is the set of all $x \in A$ with the property $P(x)$.

- Definition: $A \cap B$ is the set $\{x \in A | x \in B\}$.

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Russell's paradox again

- $\{x|P(x)\}$ is a **class**. $x \in \{x|P(x)\}$ is equivalent to $P(x)$.
- $\{x|P(x)\}$ is a **proper class**, if it is not a set.
- Fact: $\{x|x \notin x\}$ is a proper class.
- Formally, $\neg\exists y \forall x(x \in y \leftrightarrow x \notin x)$ (is provable in ZFC).

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The big V

- $V =_{\text{df}} \{x|x = x\}$.
- Fact: V is a proper class.
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The Axiom of Pair

- **The Axiom of Pair**: For any A and B, there is a set C such that $x \in C$ if and only if $x = A$ or $x = B$.
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- Definition: $\{A, B\}$ is defined as the (unique) set C in the Axiom of Pair.

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The Axiom of Union

- **The Axiom of Union**: For any set S, there exists a set U such that $x \in U$ if and only if $x \in A$ for some $A \in S$.
- Formally, …
- Definition: $\bigcup S$ is defined as the (unique) set U in the Axiom of Union.
- Definition: $A \cup B =_{\text{df}} \bigcup\{A, B\}$.

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Subset

- Definition: $A \subseteq B =_{\text{df}} \forall x(x \in A \to x \in B)$.

- $\forall x(\varnothing \subseteq x)$.
- The Axiom of Extension again: $\forall x \forall y(x \subseteq y \land y \subseteq x \to x = y)$.
- If $A \in S$, then $A \subseteq \bigcup S$.

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Ordered pairs

$$(a, b) =_{\text{df}} \{\{a\}, \{a, b\}\}.$$

$$(a, b, c) =_{\text{df}} ((a, b), c).$$

$$(a, b, c, d) =_{\text{df}} (((a, b), c), d)$$

Fact:

- $(a, b) = (a', b')$ iff $a = a'$ and $b = b'$.
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Cartesian products

$$A \times B =_{\text{df}} \{(a,b) | a \in A, b \in B\}.$$

$$A \times B \times C =_{\text{df}} (A \times B) \times C$$

Fact:

- $A \times B = \{(a,b) \in \mathscr{P}(\mathscr{P}(A \cup B)) | a \in A, b \in B\}.$
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Relations

- A set R is a **binary relation**, if it is a subset of $A \times B$ for some A and B.
- Or equivalently, A set R is a binary relation if all elements of R are ordered pairs. We will use xRy instead of $(x, y) \in R$.

$$\text{dom}R =_{\text{df}} \{x | \exists y(xRy)\}$$

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Functions

- A binary relation F is called a **function** (or **mapping**, **correspondence**) if aFb and aFb' imply $b = b'$ for any a, b, and b'.
- For any $a \in \mathrm{dom}F$, we use $F(a)$ to denote the unique element b such that aFb, which is called the **value of F at a**.
- F is a function **from A to B**, if $\mathrm{dom}F = A$ and $\mathrm{ran}F \subseteq B$. Notation: $F : A \to B$.

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Some special functions

- A function F is an **injection** if for any $a \in \text{dom}F$ and $a' \in \text{dom}F$, if $a \neq a'$, then $F(a) \neq F(a')$.
- F is a function from A **onto** B, if $\text{dom}F = A$ and $\text{ran}F = B$.
- An injection from A onto B is called a **bijection from A to B**, or a **one-to-one correspondence from A to B**.

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Thanks for your attention!

Q & A