一、大会报告

Spectrum of the 1–Laplacian and Cheeger's constant on graphs

Kung Ching Chang

Peking University, Beijing 100871, China

We develop a nonlinear spectral graph theory, in which the Laplace operator is replaced by the 1- Laplacian. The eigenvalue problem is to solve a nonlinear system involving a set valued function. In the study, we investigate the structure of the solutions, the minimax characterization of eigenvalues, the multiplicity theorem, etc. The graphic feature of eigenvalues are also studied. In particular, Cheeger's constant, which has only some upper and lower bounds in linear spectral theory, equals to the first non-zero eigenvalue for connected graphs. An algorithum of Cheeger's cut, based on the above characterization is also studied.

Groups and graphs

Yanquan Feng

Beijing Jiaotong University, Beijing 100044, China

In this talk, we sketch the past and present situations of groups and graphs at home and abroad. Some hot research topics, including conjectures and problems, in groups and graphs are introduced. The main research topics presented in this talk are automorphism groups of symmetric graphs, vertex-transitive non-Cayley graphs and symmetric graphs, where the first concentrats on Weiss conjecture, CI-problem and normality of Cayley graphs.

Ramsey functions and probabilistic method

Yusheng Li

Tongji University, Shanghai 200092, China

This talk is on estimating the Ramsey functions by probabilistic method. To find good bounds for such functions, many new ideas, techniques and methods have developed. Among these, probabilistic method, semi-random methods, random process, and algebraic method are often used. Recently, quasi-random method is used. Note that many algebraic graphs are quasi-random in some way.

Graph partitions: progresses and problems

Baogang Xu

Nanjing Normal University, Nanjing, China

Graph partition problem is one of the most important topic in structural graph theory, since many problems in graph theory can be treated as a partition of the vertices into sets with some preassigned properties. In this talk, we will present some results and problems on graph partitions originated from the Max-Cut problem.

Graph theory in fullerenes

Heping Zhang

School of Mathematics and Statistics, Lanzhou University, Lanzhou, China

The discovery of fullerenes greatly expanded the number of known carbon allotropes. A *fullerene graph* is the molecular graph of a fullerene, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds between pairs of atoms. In general, a fullerene graph is a planar (or spherical) cubic graph with exactly 12 pentagonal faces and other hexagonal faces. In recent years, mathematical researches on fullerene graphs have been made extensively. Some progresses are reviewed in this talk, such as enumerations for fullerene isomers and perfect matchings, Hamilton cycles, isolated pentagon rule, leapfrog fullerenes and complete cover, Clar number and extremal fullerenes, k-resonant fullerenes, etc.

二、邀请报告

Acyclic and generalized acyclic coloring of graphs

Jiansheng Cai

Weifang University, Weifang, China

A vertex coloring of a graph G is called r-acyclic if it is a proper vertex coloring such that every cycle D receives at least $\min\{|D|, r\}$ colors. The r-acyclic chromatic number of G is the least number of colors in an r-acyclic coloring of G. We prove that for any number $r \geq 4$, the r-acyclic chromatic number of any graph G with maximum degree $\Delta \geq 11$ and with girth at least $(r-1)\Delta$ is at most $(4r-3)\Delta$.

Some extremal problems and results on the existence of trees

Yaojun Chen

Nanjing University, Nanjing, China

It is well known that any graph with minimum degree at least k must contain all trees of order $k+1$. In this topic, we will discuss some extremal problems and results concerning the existence of all trees in a graph, and the relations among these problems.

Some research progress on spectrally arbitrary patterns

Yubin Gao

Department of Mathematics, North University of China, Taiyuan, Shanxi

This talk will introduce some research progress on spectrally arbitrary sign patterns, ray patterns and complex sign patterns.

Distance and Laplacian matrices of graphs

Yaoping Hou

Department of Mathematics, Hunan First Normal University, Changsha, Hunan 410081, China

Graham, Lovász, et al. proved a very attractive theorem about the determinant of the distance matrix $D(G)$ of a strongly connected directed graph G as a function of the distance matrix of its blocks, and gave the formula for the inverse of the distance matrix of a tree. There are many extensions for results of Graham and Lovász.

In this talk, we give a survey on determinants, Smith normal forms and the inverses of distance and q-distance matrices, and the connections between distance and Laplacian matrices on graphs or digraphs.

The main eigenvalue problems of graphs

Qiongxiang Huang

Xinjiang University, Xinjiang, China

Let G be a graph with adjacency matrix A and diagonal degree matrix D. The eigenvalues of G are the eigenvalues of A. Let $\mathcal{E}_A(\lambda) = \{x \in R^n \mid Ax = \lambda x\}$ be the eigenspace of λ. We call λ a main eigenvalue of G if there exists $x \in \mathcal{E}_A(\lambda)$ such that x is not orthogonal to all-one vector e, i.e., $x^T e \neq 0$. Otherwise, λ is a non-main eigenvalue. The Laplacian and Signless Laplacian eigenvalues of G are, respectively, the eigenvalues of Laplacian matrix $L = D - A$ and Signless Laplacian matrix $L = D + A$. Their main and non-main eigenvalues are similarly defined.

This report summarizes some known and new results. It contains the following contents.

1. the main eigenvalues of graphs and some related properties.
2. the problem to characterize the graphs that have exactly k main A-eigenvalues, known and some new results.
3. the graphs that have exactly two main Q-eigenvalues.
4. classification of graphs according to their main eigenvalues and some problems related to main eigenvalues.

Link polynomials: computation, zeros, and beyond

Xian'an Jin

School of Mathematical Sciences, Xiamen University, Xiamen, Fujian 361005, China

In the talk we restrict ourselves to three classical knot polynomials: The Jones polynomial, The HOMFLY polynomial and the Kauffman polynomial. The Jones polynomial is an invariant of oriented links discovered by Vaughan F. R. Jones in 1984 who won the Fields medal in 1990 for it. The HOMFLY and Kauffman polynomials are both generalizations of the Jones polynomial.

There are several ways to build the connection between links and graphs, including a classical one-to-one correspondence between link diagrams and signed plane graphs known more than a century of years ago at the early stage of knot theory. Consequently, the relation between link polynomials and graph polynomials is established. For example, F. Jaeger built a relation between the HOMFLY and Tutte polynomials. In addition, graph polynomials are closely related to the Potts model partition functions in statistical mechanics. For example, the chromatic polynomial is the zero-temperature partition function of the q-state Potts antiferromagnet.

Computing link and graph polynomials mentioned above are all very difficult in general. In this talk, we first present a formula for computing the Tutte polynomial of the signed graph formed from a labeled graph by edge replacements in terms of the chain polynomial of the labeled graph which was introduced by R. C. Read and E. G. Whitehead Jr. in 1999. Motivated by its connection to statistical mechanics, we study zeros of the Jones polynomial, presenting two theoretical results: (a) limits of zeros of Jones polynomials of link (diagrams) corresponding to homeomorphic plane graphs are the unit circle centered at the origin and several isolated points; (b) zeros of Jones polynomials of pretzel knots are dense in the whole complex plane. We then try to extend above results from the Jones polynomial to HOMFLY and Kauffman polynomials. Finally, we apply our results to DNA and protein polyhedron found or synthesized in chemistry and molecular biology.

This talk is a summation of t papers collaborated with Professors Fuji Zhang, Fengming Dong etc.

References:

[1] Xian'an Jin, Fuji Zhang, The Kauffman brackets for equivalence classes of links, Advances in Applied Mathematics 34 (2005) 47-64.

[2] Xian'an Jin, Fuji Zhang, The replacements of signed graphs and Kauffman brackets of link families, Advances in Applied Mathematics 39 (2007) 155-172.

[3] Xian'an Jin, Fuji Zhang, Fengming Dong, Eng Guan Tay, Zeros of the Jones polynomial are dense in the complex plane, The Electronic Journal of Combinatorics 17 (2010) 1:R94.

[4] Xian'an Jin, Fuji Zhang, Zeros of the Jones polynomial for multiple crossing-twisted links, Journal of Statistical Physics 140 (2010) 6:1054-1064.

[5] Xian'an Jin, Fuji Zhang, On the location of the Homfly polynomial, Journal of Statistical Mechanics: Theory and Experiment (2011) P07011.

[6] Xian'an Jin, Fuji Zhang, The architecture and the Jones polynomial of polyhedral links, Journal of Mathematical Chemistry 49 (2011) 2063-2088.

[7] Xian'an Jin, Fuji Zhang, The Homfly and dichromatic polynomials, Proceedings of the American Mathematical Society 140 (2012) 1459-1472.

[8] Xian'an Jin, Graphical calculi for the Dubrovnik polynomial with applications,Journal of Knot Theory and its Ramifications 21 (2012) 10:1250100, 21 pages.

[9] Xiao-Sheng Cheng, Xian'an Jin, The braid index of complicated DNA polyhedral links,PLoS One 7 (2012) 11: e48968.

[10] Jun Ge, Xian'an Jin, The Homfly polynomial of classical pretzel links and its application to chirality, Journal of Knot Theory and its Ramifications 22 (2013) 13:1350076

Anti–Ramsey numbers for matchings in graphs

Zemin Jin

Department of Mathematics, Zhejiang Normal University

Anti-Ramsey number was introduced by Erdős *et al.* in 1973. The anti-Ramsey number $AR(K_n, H)$ for the graph H in K_n is defined to be the maximum number of colors in an edge coloring of K_n without rainbow H. It has been shown that the anti-Ramsey number $AR(K_n, H)$ is closely related to Turán number $ex(n, \mathcal{H})$ of the family $\mathcal{H} = \{H - e : e \in E(H)\}$ in K_n , i.e., $AR(K_n, H) - ex(n, \mathcal{H}) = o(n^2)$. It follows from the asymptotic of Turán numbers that $AR(K_n, H)/\binom{n}{2} \to 1 - (1/d)$ as $n \to +\infty$, where $d + 1 = \min\{\chi(H - e) : e \in E(H)\}$. So the anti-Ramsey number $AR(K_n, H)$ is determined asymptotically for graphs H with $\min\{\chi(H - e) : e \in E(H)\} \geq 3$. The case $\min\{\chi(H - e) : e \in E(H)\} = 2$ still remains harder. Also, the bipartite version of the anti-Ramsey number was introduced and studied during the last decades. Besides this, the anti-Ramsey numbers for several special graph classes in complete and complete bipartite graphs have been determined exactly. Firstly, Li and Xu considered the anti-Ramsey number for matchings in a general bipartite graph.

In this talk, we will present some our results on the anti-Ramsey numbers for matchings in complete graphs, complete graphs and regular bipartite graphs.

Extremal graph theory based on the p–spectral radius

Liying Kang

Department of Mathematics, Shanghai University, Shanghai, 200444, China

The p–spectral radius of graphs unifies several graph parameters like the number of edges and the spectral radius of the adjaceny matrix.

This talk presents solutions to some classical extremal problems using the p–spectral radius instead of the number of edges. These results merge large parts of classical and spectal extremal graph theory into one. The results are joint work with Xiying Yuan and V. Nikiforov.

On multiple colorings of graphs

Wensong Lin

Department of Mathematics, Southeast University, Nanjing 210096, P.R. China

Let t be a positive integer and S a set of integers. If for any two elements a and b of S, $|a-b| \geq t$, then S is said to be t-separated. For two sets I and J of integers, the distance between I and J, denoted by $d(I,J)$, is defined as $\min\{|i-j| : i \in I, j \in J\}$.

Let n, t be two positive integers and $j_1, j_2, \ldots, j_m$ be m nonnegative integers. An n-fold t-separated $L(j_1, j_2, \ldots, j_m)$-labeling of a graph G is an assignment f of sets of nonnegative integers of order n to the vertices of G such that the following two conditions are satisfied: (1) for any vertex v, $f(v)$ is t-separated; (2) for any two vertices u and v, if $d_G(u,v) = i$ ($i \in \{1, 2, \ldots, m\}$), then $d(f(u) - f(v)) \geq j_i$, where $d_G(u,v)$ is the length (number of edges) of a shortest path between u and v in G. Integers assigned to vertices by f are called labels. The difference between the maximum and minimum labels used by f is called the span of f. The n-fold t-separated $L(j_1, j_2, \ldots, j_m)$-labeling number of G, denoted by $\lambda_t^n(j_1, j_2, \ldots, j_m)(G)$, is defined as the minimum span over all n-fold t-separated $L(j_1, j_2, \ldots, j_m)$-labelings of G.

Channel assignment problem can be formulated as n-fold t-separated $L(j_1, j_2, \ldots, j_m)$-labelings problem.

In this talk, we discuss several special cases of n-fold t-separated $L(j_1, j_2, \ldots, j_m)$-labelings of graphs.

Path covering number and $L(2,1)$–labeling number of graphs

Changhong Lv

Department of Mathematics, East China Normal University, Shanghai, 200241, China

A *path covering* of a graph G is a set of vertex disjoint paths of G containing all the vertices of G. The *path covering number* of G, denoted by $P(G)$, is the minimum number of paths in a path covering of G. An k-$L(2,1)$-*labeling* of a graph G is a mapping f from $V(G)$ to the set $\{0,1,\ldots,k\}$ such that $|f(u)-f(v)| \geq 2$ if $d_G(u,v)=1$ and $|f(u)-f(v)| \geq 1$ if $d_G(u,v)=2$. The $L(2,1)$-*labeling number* $\lambda(G)$ of G is the smallest number k such that G has a k-$L(2,1)$-labeling.

The purpose of this talk is to introduce our recent results on path covering number and $L(2,1)$-labeling number of graphs. Our main work extends most of results in [On island sequences of labelings with a condition at distance two, Discrete Applied maths 158 (2010), 1-7] and can answer an open problem in [On the structure of graphs with non-surjective $L(2,1)$-labelings, SIAM J. Discrete Math. 19 (2005), 208-223]. This is a joint work with Qing Zhou.

Maximizing proper colorings on graphs

Jie Ma

The University of Science and Technology of China

An old problem of Linial and Wilf asks for the graphs with n vertices and m edges which maximize the number of proper q-colorings on vertices. Loh, Pikhurko and Sudakov asymptotically reduced the problem to an optimization problem. We show that for any instance, the optimization problem always has a solution which corresponds to either a complete multipartite graph or a graph obtained from complete multipartite graph by removing two incident edges. We then apply this structural result to general instances, including a conjecture of Lazebnik from 1989 which asserts that for any $q \geq s \geq 2$, the Turán graph $T_s(n)$ has the maximum number of proper q-coloring among all graphs with the same numbers of vertices and edges. We disprove this by providing infinity many counterexamples (s,q) for $s < q \leq \Theta(s^{3/2})$. On the other hand, we show that, when $q \geq \Theta(s^2)$, Turán graph $T_s(n)$ indeed achieves the maximum.

This is a joint work with Humberto Naves.

The t–coefficient method, symbolization of summation formulae, and inverse q–operators with applications to q–series

Xinrong Ma

Department of Mathematics, Suzhou University

A general principle for symbolization of summation and transformation for q–series is sketched via the use of the t–coefficient method and symbolic operator approach. According to such principle, the inverses of two pairs of the q–operators are obtained. As applications to q–series, some new transformations of q–series are presented.

On the Laplacian spectral radius of c–cyclic graph and upper bounds of the kth Laplacian eigenvalue of graph

Haiying Shan

Tongji University, Shanghai 200092, China

Let $\mathbb{C}(n, \Delta; c)$ be the class of c–cyclic graphs with maximum degree Δ and n vertices. In this talk, we characterize all the extremal graphs that achieve the smallest Laplacian spectral radius of $\mathbb{C}(n, \Delta; c)$ for $0 \le c \le \frac{(\Delta-1)(\Delta-2)}{2}$. We also characterize the unique extremal graph that achieves the largest Laplacian spectral radius of $\mathbb{C}(n, \Delta; 1)$ when $\Delta \ge \lceil \frac{n+2}{3} \rceil$ and $d_4 \ge 2$ or $\Delta \ge \lceil \frac{n}{3} \rceil + 1$ and $d_4 = 1$, respectively. We also prove that for two unicyclic graphs G and G' on n vertices, if $\Delta(G) \ge \lceil \frac{11n}{30} \rceil + 2$ and $\Delta(G) > \Delta(G')$, then $\lambda(G) > \lambda(G')$, and the bound "$\lceil \frac{11n}{30} \rceil + 2$" is best possible.

Further more, we obtain two new upper bounds of the kth (signless) Laplacian eigenvalue of graphs with n vertices:

1. $\mu_i(G) \le i - 1 + \min_{U_i} \max\{|N_H(v_k) \cup N_H(v_j)| : v_k v_j \in E(H)\}$.
   where $N_H(v_k)$ is the set of neighbors of vertex v_k in $V(H) = V(G) \backslash \{U_i\}$, U_i is any $(i-1)$–subset of $V(G)$

2. $\max\Big\{\mu_i(G),\, q_i(G)\Big\} \le \max\limits_{i \le k \le n} \left\{ d_G(v_k) + \sum\limits_{v_j \in N_G(v_k) \cap N} \frac{d_G(v_j)}{d_G(v_k)} \right\} \le 2 d_G(v_i)$,
   where $d_G(v_i)$ is the i-th largest degree of G and $N = \{v_i,\, v_{i+1}, \ldots,\, v_n\}$.

This work is joint with Muhuo Liu.

New results on the eigenvalues of distance matrix of graphs

Jinlong Shu

East China Normal University

In this talk, we will introduce some results on the eigenvalues of distance matrix, such as, sharp upper and lower bounds on the distance spectral radius, which connected graphs are determined by their distance eigenvalues, distance spread, distance energy and such on. Furthermore, we will list some open problems which are related to the distance eigenvalues.

Equitable symbol weight codes

Chengmin Wang

School of Science, Jiangnan University, Wuxi 214122, China

Power line communications is a technology that enables the transmission of data over electric power lines. However, power lines present a difficult communications environment and overcoming permanent narrowband disturbance has remained a challenging problem.

In this talk, we shall introduce a new parameter that more precisely captures a code performance against permanent narrowband noise. As a result, a new class of codes, namely, equitable symbol weight codes, is defined, which are optimal with respect to this measure. In addition, some constructions of optimal equitable symbol weight codes are presented.

Some techniques in graph colorings

Guanghui Wang

School of Mathematics, Shandong University, Jinan 250100,China

Graph coloring is an old subject with many important applications. Variants of graph coloring are not only important in their various applications, but also they have given rise to some very interesting mathematical challenges and open questions.

In this talk, we will introduce some techniques in graph colorings. We will mainly focus on the generalized acyclic edge colorings and the so-called entropy compression method.

Locally conditions for a graph to be Hamilton–(connected)

Liming Xiong

Department of Mathematics, Beijing Institute of Technology, Beijing 100081, China

A parameter is *locally* if it involves an induced subgraph of G that is isomorphic to a given graph H (for example, minimum degree, and so on); *global*, otherwise (for example, minimum degree sum of an independent set, connectivity, independence number and so on). Similarly, a property of a graph G is called *locally* if it involves an induced subgraph of G that is isomorphic to a given graph H (for example, forbidden subgraph, and so on); *global*, otherwise (for example, bipartite graphs (without odd cycle) and so on). Those parameter or property guaranteeing a graph to be hamiltonian is called to be *locally condition*. A classic parameter or property guaranteeing a graph to be hamiltonian often makes those graphs to have many edges (these graph is called *dense*).

A generally problem is how to make a global condition to be locally. Some papers worked on this topic. For example, the well-known Chvátal-Erdös theorem says that every graph such that its independence number is not larger than its connectivity is hamiltonian. A. Saito considered the locally Chvátal-Erdös condition whether a graph with a similar locally property (i.e., every closed neighbors of a vertex satisfies the Chvátal-Erdös condition) is hamiltonian.

In this talk, we shall present some locally conditions guaranteeing a graph to be hamiltonian (or hamiltonian connected).

Melham's conjecture on sums of odd powers of Fibonacci numbers

Arthur L. B. Yang

Center for Combinatorics, Nankai University

Let F_n denote the nth Fibonacci number and L_n denote the nth Lucas number. Melham conjectured that for any $n, m \geq 1$, the sum

$$L_1 L_3 L_5 \cdots L_{2m+1} \sum_{r=1}^{n} F_{2r}^{2m+1}$$

can be expressed as $(F_{2n+1} - 1)^2 P_{2m-1}(F_{2n+1})$, where $P_{2m-1}(x)$ is a polynomial of degree $2m - 1$ with integer coefficients. Based on a formula due to Prodinger, we give an affirmative answer to Melham's conjecture.

This is a joint work with Brian Y. Sun and Matthew H.Y. Xie.

An independent set approach to the lower bound of some codes

Yiting Yang

Tongji University

Each code can be characterized as an independent set of a graph or a hypergraph. By applying some results on the lower bound of independent number, we improve the lower bounds of permutation code, perfect hash family, frameproof code and separable code under some parameters respectively.

Enomoto–Katona codes and Armstrong codes from relational database systems

Xiande Zhang

Nanyang Technological University

I will introduce two types of code design problems of some practical interest in understanding complexities of dependencies in relational database systems. One problem is the construction of optimal codes in a new metric space, called the Enomoto–Katona space, which measures the difference between two unordered pairs of disjoint subsets. Enomoto–Katona space has recently been considered in connection with the study of implication structures of functional dependencies and their generalizations in relational databases. The other code is called Armstrong code, whose minimum distance is attained in "every possible direction", that is if the minimum distance is d, then for every possible coordinate subset of size d, there exist two codewords that differ in exactly those positions. Armstrong codes were introduced when studying Armstrong instances of databases of bounded domains. Recent progress on these two problems will be discussed.

Nodal domain partition and community structure in networks

Xiaodong Zhang

Department of Mathematics, Shanghai Jiao Tong University, Shanghai, 200240, China.

It is difficult to detect and evaluate the number of communities in complex networks, especially when the situation involves an ambiguous boundary between the inner- and inter-community densities. In this paper, Discrete Nodal Domain Theory is used to provide a criterion to determine how many communities a network has and how to partition these communities by means of the topological structure and geometric characterization. By capturing the signs of the Laplacian eigenvectors, we separate the network into several reasonable clusters. The method leads to a fast and effective algorithm with application to a variety of real networks data sets.

This work is jointed with Lei Gu and Bian He.

Some aspects of spectral graph theory

Bo Zhou

Department of Mathematics, South China Normal University, Guangzhou 510631, P.R. China

We survey some of our results in spectral graph theory, including some aspects on adjacency, Laplacian, signless Laplacian and distance spectra.

Bounds and constructions of t–SEEDs

Junling Zhou

Institute of Mathematics Beijing Jiaotong University, Beijing 100044, China

Quantum jump codes are quantum codes which correct errors caused by quantum jumps. A spontaneous emission error design (SEED) was introduced by T. Beth et al. in 2003 to construct quantum jump codes.

In this talk, the bounds on the dimension m of t–$(v, k; m)$ SEEDs will be discussed, together with some combinatorial constructions for such designs.

超大规模集成电路物理设计自动化中的电路划分和布局

朱文兴

福州大学离散数学与理论计算机科学研究中心，福州 350108

超大规模集成电路物理设计主要分为电路划分，版图规划，布局和布线等阶段，其中每个阶段的数学模型都是 NP 困难的组合优化问题。随着超大集成电路制造工艺的迅速发展，现在已能生产 20nm 的集成电路芯片，一个芯片上集成了高达数十亿个电路器件。因此这些 NP 困难组合优化问题是超大规模问题，给求解算法的设计带来了巨大的挑战，也对超大规模集成电路计算机辅助设计工具提出了更高的要求。

该报告将简要介绍超大规模集成电路物理设计自动化的基本流程，集成电路划分问题和优化算法，集成电路布局问题和优化算法，以及我们近期的部分主要研究工作。

三、小组报告

Some infinite classes of partitioned difference families

Jingjun Bao

Department of Mathematics, Suzhou University, Suzhou 215006, China

Let $F = \{D_0, D_1, \ldots, D_{l-1}\}$ be a (v, K, λ) difference family (DF) over an abelian group G. If F forms a partition of G, then it is called partitioned and denoted by (v, K, λ)-PDF. Each $(n, \{|D_0|, |D_1|, \ldots, |D_{l-1}|\}, \lambda)$-PDF can be identified with a zero difference balanced function f with parameters (n, l, λ) from $(G, +)$ to $(B, +)$ defined by $f(x) = b_i$ for $x \in D_i$, $0 \leq i \leq l-1$. Zero difference balanced functions were first introduced by Ding (2008) in constructing optimal constant composition codes, optimal and perfect difference systems of sets.

In this talk, we will talk about several constructions of partitioned difference families.

An implicit degree condition for pancyclicity of graphs

Junqing Cai

School of Mathematics and Information Science, Henan Polytechnic University

A graph G is called pancyclic if it contains cycles of length l, for each $3 \leq l \leq n$. In 1989, Zhu, Li and Deng introduced the definition of implicit degrees of a vertex v in a graph G, denoted by $id(v)$, by using the degrees of the vertices in its neighborhood and the second neighborhood. And they obtained sufficient conditions with implicit degrees for a graph to be Hamiltonian. In this paper, we prove that if G is a 2-connected graph of order $n \geq 3$ such that $id(v) \geq n/2$ for each vertex v of G, then G is pancyclic unless G is bipartite, or else $n = 4r$, $r \geq 2$ and G is in a class of graphs F_{4r}.

Average length of the longest k–alternating subsequence

Tommy Wuxing Cai

School of Sciences, South China University of Technology, Guangzhou 510640, China

We prove a conjecture of D. Armstrong on the average maximal length of k-alternating subsequence of permutations. The $k = 1$ case is a well-known result of Richard Stanley.

Formal solutions of q–difference equations and some problems of integrals and generating functions

Jian Cao

Department of Mathematics, Hangzhou Normal University, Hangzhou 310036, China.

Two q-difference equations with solutions expressed by q–exponential operator identities are investigated. As applications, two extensions of Ramanujan's formulas for q–beta integral are given, two generalizations of Andrews–Askey integral are obtained. In addition, generating functions for generalized Al–Salam–Carlitz polynomials are deduced. At last, a generalized transformation identity is gained.

The orbits of the oaces of a weight polytope under the action of Weyl group

Zhuo Li[1], You'an Cao[1], Zhenheng Li[2]

[1] Department of Mathematics, Xiangtan University, Xiangtan, Hunan, 411105, China

[2] Department of Mathematical Sciences, University of South Carolina Aiken, Aiken, SC 29801, USA

Let λ be a dominant weight and let $\Phi(\lambda)$ be the saturated weight set with highest weight λ. The weight polytope P_λ is defined as the convex hull of $\Phi(\lambda)$. This paper provides an elementary proof that there is a natural bijection between the set of orbits of weight polytope under action of Weyl group and set of parabolic subgroups of Weyl group. The bijection, now called Putcha–Renner recipe, is found by Putcha and Renner in linear algebraic monoid theory.

Resolution of indecomposable integral flows on signed graphs

Beifang Chen[1], Jue Wang[2], Thomas Zaslavsky[3]

[1]Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

[2]Department of Mathematics and Physics, Shenzhen Polytechnic, Shenzhen, Guangdong Province, 518088, P.R. China

[3]Department of Mathematical Sciences, Binghamton University (SUNY), Binghamton, NY 13902, U.S.A.

It is well-known that each nonnegative integral flow of a directed graph can be decomposed into a sum of nonnegative graph circuit flows, which cannot be further decomposed into nonnegative integral sub–flows. This is equivalent to saying that indecomposable flows of graphs are those graph circuit flows. Turning from graphs to signed graphs, the indecomposable flows are much richer than that of ordinary unsigned graphs. The present paper is to give a complete description of indecomposable flows of signed graphs from the viewpoint of resolution of singularities by introducing covering graphs.

A characterization of $L(2,1)$–labeling number for trees with maximum degree 3

Dong Chen

Zhejiang Normal University

An $L(2,1)$–labeling of a graph is an assignment of nonnegative integers to the vertices of G such that adjacent vertices have numbers at least 2 apart, and vertices at distance 2 have distinct numbers. The $L(2,1)$–labeling number is the minimum range of labels over all such labeling. It was shown by Griggs and Yeh [Labelling graphs with a condition at distance 2, SIAM J. Discrete Math. 5(1992)586-595] that the $L(2,1)$–labeling number of a tree is either $\Delta+1$ or $\Delta+2$.

In this paper, we give a complete characterization of $L(2,1)$–labeling number for trees with maximum degree 3.

List vertex–arboricity of toroidal graphs

Min Chen, Li Huang, Weifan Wang

Department of Mathematics, Zhejiang Normal University, Jinhua 321004, China

The vertex-arboricity $a(G)$ of a graph G is the minimum number of colors required to color the vertices of G such that no cycle is monochromatic. The list vertex-arboricity $a_l(G)$ is the list–coloring version of this concept. Kronk and Mitchem (1975) proved that every toroidal graph G without 3–cycles has $a(G) \leq 2$. Choi and Zhang (2014) proved that every toroidal graph G without 4–cycles has $a(G) \leq 2$. Borodin and Ivanova (2009) proved that every planar graph G without 4-cycles adjacent to 3-cycles has $a_l(G) \leq 2$.

In this talk, we will first show some known results on this direction. Then, we will present our recent result that every toroidal graph G without adjacent 4– and 3–cycles has $a_l(G) \leq 2$. This improves and extends all these above results.

Weakly connectedness of tensor product of digraphs

Sheng Chen

Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China

In this talk, I will present a characterization for digraphs whose tensor product is weakly connected, which solves an open problem posed by Harary and Trauth (1966).

Total positivity of Riordan arrays

Xi Chen

Dalian University of Technology

In this talk, we present sufficient conditions for total positivity of Riordan arrays. As applications we show that many well-known combinatorial triangles are totally positive and many famous combinatorial numbers are log-convex in a unified approach.

The extreme eigenvalues and maximum degree of k–connected irregular graphs

Xiaodan Chen

College of Mathematics and Information Science, Guangxi University,
Nanning 530004, Guangxi, P.R. China

Let $\lambda_1(G)$ be the largest eigenvalue and $\lambda_n(G)$ be the smallest eigenvalue of a k–connected irregular graph G with n vertices, m edges and maximum degree Δ.

It is proved that

$$\Delta - \lambda_1(G) > \frac{(n\Delta - 2m)k^2}{(n\Delta - 2m)[n^2 - 2(n-k)] + nk^2}.$$

This inequality improves previous results of several authors and implies two lower bounds on $\Delta + \lambda_n(G)$ which also refine some known bounds. Another lower bound on $\Delta - -\lambda_1(G)$ for a connected irregular graph G is given as well.

Log–concave conjecture for directed genus distribution

Yichao Chen

College of Mathematics and Econometrics, Hunan University, Changsha 410082, China

A digraph embedding (Tutte called it alternating dimap) is an Eulerian embedding D which is oriented so that the edges around each vertex are directed alternately into and out of that vertex. The directed genus distribution of the digraph D is the sequence

$$g_0(D),\ g_1(D),\ g_2(D),\ \ldots,$$

where $g_i(D)$ is the number of cellular directed embeddings of D on the surface S_i. A well–known conjecture in topological graph theory, which says that all genus distributions of digraphs (graphs) are log–concave.

In this talk, we shall present some new results on log–concave conjecture. Our recent results are obtained by the newly developed tool, which are called transfer (or production) matrix.

Reference:

[1] W. T. Tutte, The dissection of equilateral triangles into equilateral triangles, Proc. Cambridge Philos. Soc. 44 (1948) 463 – 482.

[2] W. T. Tutte, Duality and trinity, in: Infinite and Finite Sets (Colloq., Keszthely, 1973), Vol. III, Colloq. Math. Soc. Janos Bolyai, Vol. 10, North-Holland, Amsterdam, 1975, pp. 1459 – 1472.

[3] C.P. Bonnington, M. Conder, M. Morton, and P. McKenna, Embedding digraphs on orientable surfaces, J. Combin. Theory (B) 85 (2002) 1 – 20.

[4] Y. Chen, J.L. Gross and X.D. Hu, Enumeration of digraph embeddings, European Journal of Combinatorics. 36 (2014), 660-678.

The applications of the Combinatorial Nullstellensatz on neighbor sum distinguishing colorings

Laihao Ding

School of Mathematics, Shandong University, 250100, Jinan, Shandong

Let $G = (V, E)$ be a graph and ϕ be a total (or an edge) coloring of G by using the color set $\{1, 2, \ldots, k\}$. Let $f(v)$ denote the sum of the color of the vertex v and the colors of all incident edges (or just the colors of all incident edges) of v. We say that ϕ is neighbor sum distinguishing if for each edge $uv \in E(G)$, $f(u) \neq f(v)$. The smallest number k is called the neighbor sum distinguishing chromatic number (or neighbor sum distinguishing index). Such graph coloring problems can be translated into searching for a substitute for each variable of some polynomial such that the polynomial is nonzero.

And the Combinatorial Nullstellensatz tells us such a substitute exists if the coefficient of some monomial of maximum degree in the polynomial is nonzero.

In this report, I will mainly introduce the applications of the Combinatorial Nullstellensatz on neighbor sum distinguishing colorings and some results we get in this field.

关于希伍德对顶点数用数学归纳法证明“五色定理”的探讨

董德周

深圳市发明家协会

在《四色定理普遍地证明》研究中，我发现希伍德对顶点数用数学归纳法来证明“五色定理”是错误的。本文通过图论和逻辑推理的方法，並用式子指出了希伍德套用数学归纳法来证明“五色定理”的做法存在几个原则性的錯误，从而否定了希伍德对顶点数用数学归纳法来证明“五色定理”，为《四色定理普遍地证明》打下了基础。

Proof of a conjecture on the number of spanning trees of line graphs and its generalization

Fengming Dong[1], Weigen Yan[2]

[1]Mathematics and Mathematics Education, National Institute of Education, Nanyang Technological University, Singapore 637616

[2]School of Sciences, Jimei University, Xiamen 361021, China

Let G be an arbitrary loopless connected graph.

In this paper we show that
$$t(L(S_r(G))) = \prod_{v\in V(G)} d(v)^{d(v)-2} \sum_{E'\subseteq E(G)} t(G-E')r^{|E(G)|-|V(G)|+1-|E'|} \prod_{e\in E'} (d(u_e)^{-1}+d(v_e)^{-1}),$$
where u_e and v_e are the two ends of edge e, $t(G)$ is the number of spanning trees of G, $L(G)$ is the line graph of G and for any non-negative integer r, $S_r(G)$ is the graph obtained from G by replacing each non-bridge edge e by a path of length $r+1$ connecting the two ends of e. Thus we generalize some known results on the relation between $t(L(G))$ and $t(G)$ and prove a conjecture that $t(L(S_1(G))) = k^{m+s-n-1}(k+2)^{m-n+1}t(G)$ for any graph G of order $n+s$ and size $m+s$ in which s vertices are of degree 1 and the others are of degree k.

The acyclic matrices with a maximum number of P–vertices and a P–set of maximal size

Zhibin Du[1], Carlos M. da Fonseca[2]

[1]School of Mathematics and Statistics, Zhaoqing University, Zhaoqing 526061, Guangdong, China

[2] Department of Mathematics, Kuwait University, Safat 13060, Kuwait

Cauchy interlacing theorem is a well–known theory in matrix theory, which reveals the closed interlacing relationship between the eigenvalues of real symmetric matrix and the eigenvalues of its principal submatrix.

The P–vertex and P–set of real symmetric matrices are the definitions based on the multiplicity of eigenvalue of matrices and Cauchy interlacing theorem. Let $m_A(0)$ denote the nullity of a given matrix A of order n. Set $A(\alpha)$ for the principal submatrix of A obtained by deleting the rows and columns indexed by the nonempty subset α of $\{1, 2, \ldots, n\}$. When $m_{A(\alpha)}(0) = m_A(0) + |\alpha|$, we call α a P–set of A. In particular, when $\alpha = \{i\}$, we say i is a P–vertex of A.

A real symmetric matrix A is said to be an acyclic matrix if the associated undirected graph of A is a tree. In this talk, we will present several characterizations of the trees T for which there exists a matrix A whose associated undirected graph is T and containing a maximum number of P–vertices and a P–set of maximal size, respectively.

On the non–negativity of the complete cd–index

Jiuyu Fan

College of Mathematics, Sichuan University

The complete cd–index of a Bruhat interval is a non–commutative polynomial in the variables c and d, which was introduced by Billera and Brenti. They also conjectured that the coefficients of these polynomials are non–negative. For a cd–monomial M containing at most one d, i.e., $M = c^i$ or $M = c^i dc^j (i, j \geq 0)$, Karu showed that the coefficient of M is non–negative.

In this paper, we show that when $M = dc^i dc^j (i, j \geq 0)$, the coefficient of M is non–negative.

On the spectral radius of a class of non–odd–bipartite even uniform hypergraphs

Yizheng Fan

Anhui University

In order to investigate the non–odd–bipartiteness of even uniform hypergraphs, we introduce a class of k–uniform hypergraphs G, called $(k, \frac{k}{2})$–hypergraphs, which satisfy the property: k is even, every edge e of G can be divided into two disjoint $\frac{k}{2}$–vertex sets say e_1 and e_2 and for any edge e' incident to e, $e \cap e' = e_1$ or e_2. Such graph G can be constructed from a simple graph, which is called the underlying graph of G. We show that G is non–odd–bipartite if and only if the underlying graph of G is non–bipartite. We obtain some results for the spectral radius of weakly irreducible nonnegative tensors, and use them to discuss the perturbation of the spectral radius of the adjacency tensor or signless Laplacian tensor of a $(k, \frac{k}{2})$–hypergraph after an edge is subdivided. Finally we show that among all non-odd-bipartite $(k, \frac{k}{2})$–hypergraphs with n half edges, the minimum spectral radius of the adjacency tensor (respectively, signless Laplacian tensor) is achieved uniquely for C_n when n is odd and for $C_{n-1} + e$ when n is even.

Diagonal pattern, 2–core and their application

Shishuo Fu

College of Mathematics and Statistics, Chongqing University, Huxi campus B1
Chongqing, 401331, China

We discuss the connection between the (2–)diagonal pattern and the 2–core of a given partition. Moreover, we construct a bijection between the "tail" of the diagonal pattern and partition into even parts using certain tiling of Young diagram. Then we use this bijection to establish two partition theorems, the first of which is a natural companion to Euler's classical result on partion with distinct parts and partition with odd parts.

Equivalence classes of knot colorings and minimum number of colors

Jun Ge

School of Mathematical Sciences, Xiamen University

For any link and for any modulus m we introduce an equivalence relation on the set of nontrivial m-colorings of the link. We will show that for any prime modulus the number of equivalence classes depends on the modulus and on the rank of the coloring matrix with respect to this modulus (joint work with S. Jablan, L. Kauffman and P. Lopes). For primes 11, 13 and 17, we classify all the color sets with small cardinality which are possible to form a non-trivial coloring for some link. For any link L with non-zero determinant and any prime $p \geq 17$, we prove that the minimum number of colors mod p is at least 6. Then a positive answer to a question raised by Nakamura, Nakanishi and Satoh will be given (joint work with X. Jin, L. Kauffman, P. Lopes and L. Zhang). At the end of the talk, we will mention a conjecture related to both determinant of knots and links in knot theory and the number of spanning trees in graph theory.

On the maximal energy of tree with diameter 4

Yunpeng Ge, Suonan Renqian, Bofeng Huo, Qiangqiang Diao, Chunyun Wang

Department of Mathematics, Qinghai University, Xining810008

The energy of a simple graph is defined as the sum of the absolute values of all the eigenvalues of the corresponding adjacency matrix. Let be the collection of trees with vertices. Ou finds the structure of tree which has maximal energy among all trees with center of degree and diameter 4. This paper improves the result. By using quasi-order, Coulson integral formula and related knowledge of differential calculus, the structure of the tree with maximal energy and diameter 4 is completely determined.

Potts model partition functions on two families of fractal lattices

Helin Gong, Xian'an Jin

School of Mathematical Sciences, Xiamen University, Xiamen 361005, P. R. China

The partition function of q–state Potts model, or equivalently the Tutte polynomial, is computationally intractable for regular lattices. The purpose of this paper is to compute partition functions of q–state Potts model on two families of fractal lattices. Based on their self–similar structures and by applying the subgraph–decomposition method, we divide their Tutte polynomials into two summands, and for each summand we obtain a recursive formula involving the other summand. As a result, the number of spanning trees and its asymptotic growth constant, and a lower bound of the number of connected spanning subgraphs or acyclic root-connected orientations for each of such two lattices are obtained.

On the problem of minimum skew rank

Shicai Gong, Guanghui Xu

School of Science, Zhejiang A & F University, Hangzhou, 311300, China

The minimum skew rank of a simple graph G is the smallest possible rank among all real skew-symmetric matrices whose $(i; j)$-entry is nonzero if and only if the edge joining i and j is in G. Let C be an even cycle of a perfect matching graph G. Then C is nice if $G - C$ is still a perfect matching graph. We in this paper first generalize the terminology the nice cycle to a general graph, not necessary be perfect matching. Then we will interpret that such a terminology is better to investigate the problem of the minimum skew rank of a graph.

Fault tolerance analysis of k–ary n–cubes in terms of extra connectivity

Meimei Gu

Department of Mathematics, Beijing Jiaotong University, Beijing, 100044, China

J. Fábrega and M.A. Fiol introduced the *extra connectivity* of interconnection networks, which is regarded as a generalization of the classical connectivity and provides a more accurate tool for the fault tolerance of interconnection networks. The k–ary n–cube Q_n^k, proposed by Scott and Goodman, many interconnection networks can be

viewed as subclasses of Q_n^k, including the cycle, the torus and the hypercube. In this talk, I will talk about fault tolerance analysis of k–ary n–cubes in terms of the extra connectivity.

This is a work joint with Prof. Rong-Xia Hao.

Proof of a conjecture of Mircea Merca

Victor J. W. Guo

Department of Mathematics, Shanghai Key Laboratory of PMMP, East China Normal University, Shanghai 200241, China

We prove that, for any prime p and positive integer r with $p^r > 2$, the number of multinomial coefficients such that

$$\binom{k}{k_1, k_2, \dots, k_n} = p^r, \quad \text{and} \quad k_1 + 2k_2 + \cdots + nk_n = n,$$

is given by

$$\delta_{p^r,k}\left(\left\lfloor \frac{n-1}{p^r-1} \right\rfloor - \delta_{0,\, n \bmod p^r}\right),$$

where $\delta_{i,j}$ is the Kronecker delta and $\lfloor x \rfloor$ stands for the largest integer not exceeding x. This confirms a recent conjecture of Mircea Merca.

Equivalence classes of full–dimensional 0/1–polytopes of the hypercube

Long Guo

Center for Combinatorics, LPMC-TJKLC Nankai University, Tianjin 300071, P.R. China

Let Q_n denote the n–dimensional hypercube with vertex set $V_n = \{0, 1\}^n$. A 0/1–polytope of Q_n is the convex hull of a subset of V_n. An n-dimensional 0/1–polytope of Q_n is called a full–dimensional 0/1–polytope of Q_n. In this talk, we are concerned with the enumeration of equivalence classes of full-dimensional 0/1–polytopes under the symmetries of the hypercube. Aichholzer completed the enumeration of equivalence classes of full-dimensional 0/1–polytopes for Q_4, Q_5, and those of Q_6 up to 12 vertices. We present a method to compute the number of equivalence classes of full-dimensional 0/1–polytopes of Q_n with more than 2^{n-2} vertices. We also give an argument to enumerate equivalence classes of full–dimensional 0/1–polytopes of Q_n with vertex number close to 2^{n-2}. Using our approach, we finish the counting of equivalence classes of full–dimensional 0/1–polytopes of Q_6 with more than 12 vertices.

On additive bases II

Dongchun Han

Center for Combinatorics, Nankai University, Tianjin 300071, P.R. China

Let G be an additive finite abelian group, and let S be a sequence over G. We say that S is regular if for every proper subgroup $H \subseteq G$, S contains at most $|H| - 1$ terms from H. Let $c_0(G)$ be the smallest integer t such that every regular sequence S over G of length $|S| \geq t$ forms an additive basis of G, i.e., every element of G can be expressed as a sum over a nonempty subsequence of S. We determine $c_0(G)$ for some groups including the cyclic groups, the groups of order even, the groups of rank at least five, and all the p-groups except $G = C_p \oplus C_{p^n}$ with $n \geq 2$.

Locally planar graphs are 5–paintable

Ming Han, Xuding Zhu

College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, China

A graph G is locally planar if it is embedded in a surface with large edge–width. Thomassen [Five–coloring maps on surfaces, J. Combin. Theory Ser. B, 59 (1993) 89–105] proved that every graph embedded in a fixed surface with sufficiently large edge–width is 5-colourable. DeVos, Kawarabayashi and Mohar [Locally planar graphs are 5–choosable, J. Combin. Theory Ser. B, 98 (2008), 1215–1232] strengthened this result and proved that every graph embedded in a fixed surface with sufficiently large edge–width is 5–choosable. This paper further strengthens the result to on-line list coloring and proves that every graph embedded in a fixed surface with sufficiently large edge-width is 5–paintable.

On the divisibility properties of certain binomial sums

Bing He

East China Normal University

For any integer $n > 1$, we prove that

$$2n\binom{2n}{n} \Bigg| \sum_{k=0}^{n-1}(6k+1)\binom{2k}{k}^3 2^{8(n-k-1)}$$

and

$$2n\binom{2n}{n}\Bigg|\sum_{k=0}^{n-1}(120k^2+34k+3)\binom{2k}{k}^4\binom{4k}{2k}2^{16(n-k-1)}.$$

The first divisibility result confirms a conjecture of Z.-W. Sun.

A note on almost balanced bipartitions of a graph

Xiaolan Hu

Department of Mathematics, Nanjing University

Let G be a graph of order $n \geq 6$ with minimum degree $\delta(G) \geq 4$. Arkin and Hassin (Graph partitions with minimum degree constraints, Discrete Mathematics, 190(1998), 55-65) conjectured that there exists a bipartition S, T of $V(G)$ such that $\lfloor \frac{n}{2} \rfloor - 2 \leq |S|,\ |T| \leq \lceil \frac{n}{2} \rceil + 2$ and the minimum degrees in the subgraphs induced by S and T are at least two. In this talk, we first show that G has a bipartition such that the minimum degree in each part is at least two, and then prove that the conjecture is true if the complement of G contains no complete bipartite graph $K_{3,r}$, where $r = \lfloor \frac{n}{2} \rfloor - 3$.

Edge–face coloring and entire coloring of plane graphs

Xiaoxue Hu, Weifan Wang

Department of Mathematics, Zhejiang Normal University, Jinhua, 321004, China

A plane graph $G = (V, E, F)$ is called entirely k–colorable if there exists a mapping $c : V \cup E \cup F \to \{1, 2, \cdots, k\}$ such that any two adjacent or incident elements in $V \cup E \cup F$ have different colors. If we consider coloring only for the elements in $E \cup F$, then an edge–face k–coloring of G can be similarly defined.

Let G be a plane graph with maximum degree Δ. In this talk, we show the following results: (1) G is entirely $(\Delta + 5)$–choosable; (2) If $\Delta \geq 9$, then G is entirely $(\Delta + 2)$–colorable; (3) If $\Delta \geq 10$, then G is entirely $(\Delta + 2)$–choosable; (4) If $\Delta \geq 9$, then G is edge-face $(\Delta + 1)$–choosable.

Weakly bipancyclic graphs

Zhiquan Hu, Jing Sun

Faculty of Mathematics and Statistics, Central China Normal University

We investigate the set of cycle lengths occurring in bipartite graphs with large minimum degree. A bipartite graph is *weakly bipancyclic* if it contains cycles of every even length between the length of a shortest and a longest cycle. In this talk, it is shown that if $G = (V_1, V_2, E)$ is a bipartite graph with minimum degree at least $n/3+4$, where $n = \max\{|V_1|, |V_2|\}$, then G is a weakly bipancyclic graph of girth 4. This improves a theorem of Tian and Zang [*System Sci. Math Sci.* **2** (1989), 22-31], which asserts that if G is a Hamilton bipartite graph on $2n$ ($n \geq 60$) vertices with minimum degree greater than $2n/5 + 2$, then G is bipancyclic (i.e., G contains cycles of every even length between 4 and $2n$). By combining our result with a theorem of Jackson and Li [*J. Combin. Theory Ser. B* **62** (1994), 236-258], we obtain that every 2-connected k-regular bipartite graph on at most $6k - 38$ vertices is bipancyclic.

On remoteness and proximity in graphs

Hongbo Hua

Faculty of Mathematics and Physics, Huaiyin Institute of Technology, Huai'an, Jiangsu 223003, P.R. China

The remoteness $\rho = \rho(G)$ of a connected graph G is the maximum, over all vertices, of the average distance from a vertex to all others, while the proximity $\pi = \pi(G)$ of a connected graph G is the minimum, over all vertices, of the average distance from a vertex to all others. In the present talk, we introduce some new results concerning remoteness or proximity. Some of our results resolved conjectures involving remoteness or proximity proposed by M. Aouchiche and P. Hansen.

Adjacent vertex distinguishing index of bipartite planar graphs

Danjun Huang

Zhejiang Normal University

A proper edge coloring of a graph G is called adjacent vertex distinguishing, if any pair of adjacent vertices meets distinct color sets. The adjacent vertex distinguishing index of G, denoted by $\chi'_a(G)$, is the smallest integer k such that G has a k-adjacent

vertex distinguishing edge coloring. [K. Edwards, M. Hornak and M. Wozniak, On the neighbor distinguishing index of a graph, Graphs Combin. 22 (2006) 341-350] showed that $\chi'_a(G) \leq \Delta + 1$ for any bipartite planar graph G with $\Delta \geq 12$.

In this talk, we improve their result by showing that if G is a connected bipartite planar graph with maximum degree $\Delta \geq 7$, then $\chi'_a(G) \leq \Delta + 1$.

Average size of a self-conjugate (s, t)–core partition

Haoyang Huang

Nankai University

Armstrong, Hanusa and Jones conjectured that if s, t are coprime integers, then the average size of an (s, t)–core partition and the average size of a self-conjugate (s, t)–core partition are both equal to $\frac{(s+t+1)(s-1)(t-1)}{24}$. Stanley and Zanello calculated that the average size of an $(s, s+1)$–core partition equals $\binom{s+1}{3}/2$. Based on a bijection of Ford, Mai and Sze between self-conjugate (s, t)–core partitions and lattice paths in $\lfloor \frac{s}{2} \rfloor \times \lfloor \frac{t}{2} \rfloor$ rectangle, we obtain the average size of a self-conjugate (s, t)–core partition as conjectured by Armstrong, Hanusa and Jones.

一个组合学公式的发现和二次剩余分布密度新规律的揭示

黄益如, 黄蹇, 官瑜, 杨建生, 张朝辉

上海大学数学系

Since Gauss firstly proved quadratic reciprocity law in 1796, there are almost 200 proves about it nowadays. With the great zeal to quadratic residue theorem, there are more and more applications going through many branches of mathematics. For example, the (4.4)–Ramsey graph is obtained by regarding the vertices as elements of the field of integers modulo 17, and joining two vertices if their difference is quadratic residue of 17 (either 1, 2, 4, 8, 9, 13, 15 or 16).

It has been already many research results about the quadratic residue theorem distribution properties during 200 years. But we find a new quadratic residue continuous distribution law based on wonderful $Q = 2R$ formula.

For instance, when prime $p \equiv -1 (\bmod 8)$, there are $\frac{p+1}{4}$ isolated points. That is, if k is quadratic residue, then $k+1$ and $k-1$ are quadratic non-residue; or if k is quadratic non-residue, then $k+1$ and $k-1$ are quadratic residue. There are $\frac{p+1}{4}$ such k and $\frac{p+1}{4}$ parts that every part has two adjacent quadratic residue (or quadratic non-residue) and so on. (See details in Theorem 1.4.)

组合矩阵的结构指数——组合矩阵指数的系统化

黄宇飞

广州民航职业技术学院, 广东广州, 510403

组合矩阵的指数理论是组合矩阵论中核心的研究内容, 其不仅具有重要的理论价值, 而且在计算机科学、遍历理论、通讯理论、社会学和经济学中都有广泛的应用. 注意到: 组合矩阵的各类指数无论从定义还是结论都存在内在的联系和共性, 形形式式的指数是组合矩阵幂序列其结构式的一种特殊呈现. 故我们采用数学公理化的思想建立一个统一的概念: 组合矩阵的 $\mathbb{P}$- 结构指数, 把组合矩阵的各类指数作系统化. 基于各矩阵类 $\mathcal{M}$ 的不同性质 $\mathbb{P}$, 由结构指数的观点即可导出不同的指数. 对各类指数的分析表明: 性质 $\mathbb{P}$ 在内涵上呈现出: 包含某特殊子阵的 "特征性", 幂矩阵结构变化的 "周期性", 幂序列元素个数的 "极值性"; 在形式上表现出: "存在型" 和 "任意型", "整体化" 和 "局部化" 等. 内涵和形式犹如结构指数的 "经纬线", 在组合矩阵的 "大平面" 上, 交汇出变化多彩的各类结构指数. 而每一种指数都向我们提出了这样的挑战: 指数上界的估值, 指数集的刻画, 极矩阵的描述, 以及探索指数达到某值的矩阵集.

组合矩阵的结构指数, 不仅总结性地提供了一个涵盖组合矩阵指数的理论体系, 让我们从更高的角度对各类指数展开研究, 而且将引导我们在体系的空白中开始新的探索, 开拓了指数理论研究的新视野.

Matching and matching energy

Shengjin Ji

School of Science, Shangdong University of Technology, Zibo, Shandong

Let G be a simple graph of order n and $\mu_1, \mu_2, \ldots, \mu_n$ the roots of its matching polynomial. The matching energy of G is defined as the sum $\sum_{i=1}^{n} |\mu_i|$.

In this talk, we characterize the graph having maximum matching energy among all connected graphs with order n and edge connectivity k. We also show the extremal graph with matching energy in all bicyclic graphs. Moreover, the graph having the minimal matching energy is determined in (n, m)–graphs, where $n + 2 \leq m \leq 2(n - 2)$.

This is a joint work with Xueliang Li, Hongping Ma and Yongtang Shi.

Some problems on paths and cycles

Chunhui Lai

School of Mathematics and Statistics, Minnan Normal University, Zhangzhou, Fujian 363000, CHINA

In 1976, Thomassen C. conjectures that every longest cycle in a 3-connected graph has a chord (see Bondy J. A., Murty U. S. R., Graph theory, Graduate Texts in Mathematics, 244, Springer, New York, 2008, Unsolved Problems 65). In 1979, Kotzig A. conjectures that there exists no graph in which each pair of vertices is connected by a unique path of length k ($k \geq 3$) (see Bondy J. A., Murty U. S. R., Graph theory, Graduate Texts in Mathematics, 244, Springer, New York, 2008, Unsolved Problems 64). Let G be a connected graph with p vertices and q edges and define the parameter $r = q - p + 1$. The maximum number of cycles in such a graph is denoted by $\Psi(r)$. In 1981, Entringer R. C. and Slater P. J raised the problem of determining $\Psi(r)$ (see Entringer R. C., Slater P. J, On the maximum number of cycles in a graph, Ars Combin. 11 (1981), 289–294). This talk summarizes some results on these problem and the conjectures that relate to these.

3–Hued coloring of $K_{1,3}$–free graphs

Hao Li

Department of Mathematics, Renmin University of China, Beijing 100872, China.

For positive integers k and r, an (k, r)–coloring of a graph G is a proper k–coloring of the vertices such that every vertex of degree i will be adjacent to vertices with at least $\min\{i, r\}$ different colors. The smallest integer k for which a graph has an (k, r)–coloring is the r-hued chromatic number $\chi_r(G)$. It is known that there exist families of graphs in which the difference between $\chi_r(G)$ and the classic chromatic number tends to infinity. It has been one of the main research stream problems to identify graph families in which In this the difference between $\chi_r(G)$ and the classic chromatic number is bounded. We investigate the 3-hued chromatic number of claw-free graphs and give out its best possible upper bound.

On tetravalent s–regular Cayley graphs

Jingjian Li

School of Mathematics and Information Sciences, Guangxi University, Nanning 530004, P. R. China

A graph Γ is called s–regular if $\mathfrak{d}$ acts regularly on its s–arcs. $\Gamma = Cay(G, S)$ is said to be core-free if G is core-free in some X for $G \leq X \leq \mathfrak{d}$. It is shown in this paper that if $s \leq 2$, then there exist no core-free tetravalent s–regular Cayley graphs; and for $s \geq 3$, every tetravalent s–regular Cayley graphs is a normal cover of one of 3 core–free ones. In particular, a characterization of tetravalent s–regular Cayley graphs is given.

Keywords. Cayley graph, Normal primitive, core-free.

Nowhere–zero 3–flows in Cayley graphs on some special groups

Liangchen Li

Luoyang Normal Uiversity

Tutte conjectured that every 4–edge–connected graph admits a nowhere–zero 3–flow. In this talk, we show that this conjecture is true for Cayley graphs on some special groups.

q–Derivative operator proof, for of a conjecture of Melham

Nadia N. Li

Department of Mathematics, Zhoukou Normal University, Zhoukou 466001, China

Melham conjectured two identities between Fibonacci and Lucas numbers in 1999. Subsequently, Kilic et al. showed them in 2010 by contour integration. In this paper we present a new proof using the q–derivative operator. By examining two rational functions, we further derive several identities concerning Fibonomial coefficients. Finally, two well–known summation formulae on q–series are transformed into convolution formulae on Fibonomial coefficients.

Matching preclusion for vertex–transitive graphs

Qiuli Li, Jinghua He, Heping Zhang

School of Mathematics and Statistics, Lanzhou University, Lanzhou, China

In interconnection networks, matching preclusion is a measure of robustness when there is a link failure. Let G be a graph of even order. The matching preclusion number $mp(G)$ of it is the minimum number of edges whose deletion results in a subgraph without perfect matchings. The matching preclusion of lots of networks have been investigated individually. In this paper, we consider the matching preclusion of vertex-transitive graphs. By using the structural properties of them, we obtain their matching preclusion numbers, and mainly classify the optimal solutions. As we know, many interconnection networks are vertex-transitive. Consequently, lots of previous results on the matching preclusion of particular networks can be deduced from ours.

The multicast routing in rings with costs

Weidong Li

Yunnan University, Kunming 650091, PR China

We study the problem of multicast routing on rings with cost, which is to select some of given multicast requests with different demands and embed them on a directed ring, such that the sum of the maximum congestion of the links on the ring and the total cost of the unselected multicast requests is minimized. We prove that this problem is NP-hard even if the demand is divisible, and design a 1.582- approximation algorithm for the demand divisible case and a 3-approximation algo- rithm for the demand indivisible case. As a consequence, for any $\varepsilon > 0$, we present a $(1.582+\varepsilon)$-approximation algorithm for the case where every multicast request contains exactly one sink.

Generalized measures of fault tolerance in some recursive networks

Xiangjun Li[1], Junming Xu[2]

[1]School of Information and Mathematics, Yangtze University, Jingzhou, Hubei, 434023, China

[2]School of Mathematical Sciences, University of Science and Technology of China, Wentsun Wu Key Laboratory of CAS, Hefei,230026, China

In this talk, we consider a kind of generalized measures $\kappa^{(h)}$ and $\lambda^{(h)}$ of fault tolerance in recursive interconnection networks, which contains several well–known interconnection networks such as hypercubes, star graphs, (n, k)–star graphs and bubble–sort graphs *et al.*. The results about the generalized measures of fault tolerance enhances fault–tolerant ability of the above-mentioned networks theoretically. We propose a unified approach to compute $\kappa^{(h)}$ and $\lambda^{(h)}$ in some recursive interconnection networks, and report the recent results about this topic.

Computing the Tutte polynomials of self–similar graphs

Yunhua Liao, Yaoping Hou

Department of Mathematics, Hunan Normal University, Changsha, Hunan 410081, People's Republic of China

The Tutte polynomial of a graph, or equivalently the q-state Potts model partition function, is a two–variable polynomial graph invariant of considerable importance in both combinatorics and statistical physics. Even though this invariant is NP-hard to compute in general, there are many occasions when we face the task of computing the Tutte polynomial for some families of graphs.

In this work, based on their special structure, we find recursive formulas for the Tutte polynomials of several families of self–similar graphs. Our main technique is the subgraph–decomposition method.

On least distance eigenvalues of trees, unicyclic graphs and bicyclic graphs

Hongying Lin, Bo Zhou

Department of Mathematics, South China Normal University

The connected graphs with least distance eigenvalues in $[-2.383, 0]$ and the trees with least distance eigenvalues in $(-2-\sqrt{2}, 0]$ have been known. We determine the trees with least distance eigenvalues in $[-3-\sqrt{5}, -2-\sqrt{2}]$ and the unicyclic and bicyclic graphs with least distance eigenvalues in $(-2-\sqrt{2}, -2.383)$. We also determine the trees with the kth largest least distance eigenvalues for every positive integer k up to 28, and the unicyclic and bicyclic graphs with the kth largest least distance eigenvalues respectively for any positive integer k.

Arithmetic properties of ℓ–regular bipartitions

Lishuang Lin

School of Sciences, Jimei University, Xiamen 361021, China

In this talk, I will present two infinite families of congruences for the function $B_7(n)$ and $B_{13}(n)$, where $B_\ell(n)$ denotes the number of ℓ–regular bipartitions of n. In particular, using Ramanujan's two modular equations of degree 7, we obtain that for $\alpha \geq 2$ and $n \geq 0$,

$$B_7\left(3^\alpha n + \frac{5\cdot 3^{\alpha-1}-1}{2}\right) \equiv 0 \pmod 3.$$

Based on a powerful result on a level 13 analog of Rogers–Ramanujan continued fraction, we conclude that for $\alpha \geq 2$ and $n \geq 0$,

$$B_{13}(3^\alpha n + 2\cdot 3^{\alpha-1} - 1) \equiv 0 \pmod 3.$$

Some results of bicoloured ordered trees

Chunlin Liu

Department of Mathematics and System Sciences, National, University of Defense Technology, Changsha, Hunan 410073 P. R. China

An ordered tree can be defined inductively as an unlabelled rooted tree whose principal subtrees (the subtrees obtained by removing the root) are ordered trees and have been assigned a linear order (from left to right) among themselves. Ordered trees, non-nesting partitions, RNA secondary structures and many other combinatorial objects can be enumerated by Catalan numbers.

This paper wants to show bijections between bicoloured ordered trees and several other combinatorial objects. Many closed formulae will be obtained.

Strong q-log-convexity of the Eulerian polynomials of Coxeter groups

Lily Li Liu[1], Baoxuan Zhu[2]

[1]School of Mathematical Sciences, Qufu Normal University, Qufu 273165, PR China

[2]School of Mathematical Sciences, Jiangsu Normal University, Xuzhou 221116, PR China

In this paper we prove the strong q–log–convexity of the Eulerian polynomials of Coxeter groups using their exponential generating functions. Our proof is based on the theory of exponential Riordan arrays and a criterion for determining the strong q–log–convexity of polynomials sequences, whose generating functions can be given by the continued fraction. As consequences, we get the strong q–log–convexity of the Eulerian polynomials of types A_n, B_n, their q–analogous and the generalized Eulerian polynomials associated to the arithmetic progression $\{a, a+d, a+2d, a+3d, \ldots\}$ in a unified manner.

对最小度有所约束的图的划分问题

刘木伙

华南农业大学数学系

在此报告中, 我们将介绍一些许宝刚教授和我在 “对最小度有所约束的图的划分问题” 研究方向上所取得的新的研究结果, 这些结果改进了一些在此领域中已有的研究成果.

Some results on the distance spectral characterization of graphs

Ruifang Liu[1], Jie Xue[1], Huicai Jia[2]

[1]School of Mathematics and Statistics, ZhengzhouUniversity, Zhengzhou, Henan 450001, China

[2]College of Science, Henan Institute of Engineering, Zhengzhou, Henan 451191, China

Let G be a simple connected graph of order n and $D(G)$ be the distance matrix of G. Suppose that $\lambda_1(D(G)) \geq \lambda_2(D(G)) \geq \cdots \geq \lambda_n(D(G))$ are the distance spectrum of G. The graph G is said to be determined by its D-spectrum if with respect to the distance matrix $D(G)$, any graph with the same spectrum as G is isomorphic to G.

In this talk, we will survey some results on the distance spectral characterization of graphs.

Quadratic unitary Cayley graphs of finite commutative rings

Xiaogang Liu, Sanming Zhou

Department of Mathematics and Statistics, The University of Melbourne, Parkville, VIC 3010, Australia

The *adjacency matrix* of a graph is the matrix with rows and columns indexed by its vertices such that the (i, j)-entry is equal to 1 if vertices i and j are adjacent and 0 otherwise. The *eigenvalues* of a graph are eigenvalues of its adjacency matrix. A finite r-regular graph G is called *Ramanujan* if

$$|\lambda_G| \leq 2\sqrt{r-1},$$

where $|\lambda_G|$ is the maximum in absolute value of an eigenvalue of G other than $\pm r$. The *k-th spectral moment* of a graph G with n vertices and with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ is defined as

$$s_k(G) = \sum_{i=1}^{n} \lambda_i^k,$$

where $k \geq 0$ is an integer. The *energy* of G is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

Let R be a finite commutative ring and let $R^\times$ denote its set of units. Let $Q_R = \{u^2 : u \in R^\times\}$ and $T_R = Q_R \cup (-Q_R)$. The *quadratic unitary Cayley graph* of R, denoted by $\mathcal{G}_R$, is defined as the Cayley graph on the additive group of R with respect to T_R; that is, $\mathcal{G}_R$ has vertex set R such that $x, y \in R$ are adjacent if and only if $x - y \in T_R$. Let $R = R_1 \times R_2 \times \cdots \times R_s$ be a finite commutative ring, where R_i is a local ring with maximal ideal M_i of order m_i such that $|R_i|/m_i \equiv 1 \pmod 4$ for $1 \leq i \leq s$, and let R_0 be a local ring with maximal ideal M_0 of order m_0 such that $|R_0|/m_0 \equiv 3 \pmod 4$.

In this talk, I will present how to obtain the eigenvalues of $\mathcal{G}_R$ and $\mathcal{G}_{R_0 \times R}$. Using these results, we determine the energies and spectral moments of such quadratic unitary Cayley graphs, and find out when such a graph is hyperenergetic or Ramanujan.

Two new classes of binary sequence pairs with three–level cross–correlation

X. Liu, J. Wang, D. Wu

Department of Mathematics, Guangxi Normal University, Guilin 541004, China

A pair of binary sequences is generalized from the concept of a two–level autocorrelation function of a single binary sequence. In this paper, we describe two classes of binary sequence pairs of period $N = 2q$, where q is a prime. Those classes of binary sequence pairs are based on cyclic almost difference set pairs. They have optimal three–level cross–correlation, and either balanced or almost balanced.

因子临界图的耳朵分解与最大匹配数

刘岩

华南师范大学数学科学学院

令 G 是一个连通图. 如果对任意顶点 v，$G-v$ 都有完美匹配，则称 G 是因子临界图. Lovasz 证明每一个因子临界图都有耳朵分解.

在本报告中将用因子临界图的耳朵分解解决最大匹配计数问题, 给出三类具有特殊耳朵分解的因子临界图的最大匹配计数公式, 从而说明这种分解对匹配计数问题的作用.

Complex–L matrix and its recognition

Yue Liu[1], Haiying Shan[2]

[1]College of Mathematics and Computer Science, Fuzhou University, Fuzhou, 350108, China

[2]Department of Mathematics, Tongji University, Shanghai 200092, China

A matrix is called a complex–L matrix if its complex sign pattern implies that it is of full column rank. The definition is a generalization of L–matrices from real field to complex field. The recognition problem of complex–L matrices is studied in this paper, a combinatorial characterization of complex–L matrices is given, showing that the problem of recognizing complex–L matrices is in the class co–NP–complete. It is also shown this recognition problem can be reduced to its subproblem for which each entry of the matrices is either real or pure imaginary.

An $O(n)$ time algorithm for maximal line subgraph in a Halin graph

Dingjun Lou

Department of Computer Science, Sun Yat-sen University, Guangzhou 510275, People's Republic of China

The Maximal Line Subgraph Problem is: Given a graph $G=(V,E)$ and a positive integer $K \le |E|$, is there a subset $E' \subseteq E$ such that $|E'| \ge K$ and $G'=(V,E')$ is a line graph? The Maximal Line Subgraph Problem is an NP-complete problem for general graphs. In this paper, we introduce a linear time algorithm to find a maximal line subgraph in Halin graphs. The algorithm is optimal.

The sorting index on colored permutations and even–signed permutations with Ferrers shape

Sen-Peng Eu, Yuan-Hsun Lo, Tsai-Lien Wong

Department of Mathematics, National Taiwan Normal University

The permutation group $\mathfrak{S}_n$, signed permutation group $\mathcal{B}_n$ and even-signed permutation group $\mathcal{D}_n$ are known as the Coxeter groups of type A, B and D, respectively. On $\mathfrak{S}_n, \mathcal{B}_n, \mathcal{D}_n$ Petersen [2] respectively defined the *sorting indices* sor_A, sor_B, sor_D and proved that they are Mahonian statistics (i.e., equidistributed with the corresponding length function ℓ_A, ℓ_B or ℓ_D). Recently, Chen et al. [1] extended Petersen's work to a series of joint equidistributed set-valued statistics, while Poznanović [3] showed further that the above results hold on a Ferrers board. For $\mathfrak{S}_n$, for instance, $(\ell_A, \mathrm{Rmil}, \mathrm{Lmap}, \mathrm{Lmal})$ and $(\mathrm{sor}_A, \mathrm{Cyc}, \mathrm{Lmap}, \mathrm{Lmal})$ have the same joint distribution over the permutations corresponding to arrangements of n non-attacking rooks on a fixed Ferrers board with n rows and n columns.

As the set of colored permutations $\mathcal{G}_{r,n} := C_r \wr \mathfrak{S}_n$ is a natural extension of $\mathfrak{S}_n$ $(= C_1 \wr \mathfrak{S}_n)$ and $\mathcal{B}_n$ $(= C_1 \wr \mathfrak{S}_n)$, the first part of our work is to define the sorting index on $\mathcal{G}_{r,n}$ and then generalize those type A and B results to $\mathcal{G}_{r,n}$ and to any fixed Ferrers shape. We mainly have the following

$$\sum_{\pi\in\mathcal{G}_{r,n,\mathrm{ff}}} q^{\ell(\pi)} \prod_{t=0}^{r-1}\left(\prod_{i\in\mathrm{Rmil}^{-t}(\pi)} x_{t,i} \prod_{i\in\mathrm{Lmil}^{-t}(\pi)} y_{t,i}\right) = \sum_{\pi\in\mathcal{G}_{r,n,\mathrm{ff}}} q^{\mathrm{sor}(\pi)} \prod_{t=0}^{r-1}\left(\prod_{i\in\mathrm{Cyc}^{t}(\pi)} x_{t,i} \prod_{i\in\mathrm{Lmic}^{t}(\pi)} y_{t,i}\right)$$

$$= \prod_{j=1}^{n}\Bigg(x_{0,j} + q + \cdots + q^{j-h_j-1} + \xi_{h_j=1}(y_{0,j})q^{j-h_j} + \sum_{t=1}^{r-1}\left(x_{r-t,j}q^{2j+t-2} + q^{2j+t-3} + \cdots + q^{j+h_j+t-1} + \xi_{h_j=1}(y_{r-t,j})q^{j+h_j+t-2}\right)\Bigg),$$

where the notations ff, h_j, ξ will be defined later. Finally, analogous result for $\mathcal{D}_n$ is derived as well.

References:

[1] W. Chen, G. Gong and J. Guo, The sorting index and permutation codes, *Advances in Applied Mathematics*, 50(3): 367–389, 2013.

[2] T. K. Petersen, The sorting index, *Advances in Applied Mathematics*, 47(3): 615–630, 2011.

[3] S. Poznanović, The sorting index and equidistribution of set-valued statistics over restricted permutations, J. Combin. Theory Ser. A 125 (2014), 254–272.

Distance two labeling of the edge–path–replacement of a graph

Damei Lv

Department of Mathematics, Nantong University, Nantong 210007, P.R.China

In the channel assignment problem, we need to assign frequency bonds to transmitters, if two transmitters are too close, interference will occur if they attempt to transmit on close frequencies. In order to avoid this situation, the separation of the channels assigned to them must be sufficient. Moreover, if two transmitters are close but not too close, the channels assigned must be different. This problem is known as distance two labeling of a graph. For a positive integer d, an $L(d,1)$-labeling of a graph G is an assignment of nonnegative integers to $V(G)$ such that the difference between labels of adjacent vertices is at least d, and the difference between labels of vertices that are distance two apart is at least 1. The span of an $L(d,1)$-labeling of a graph G is the difference between the maximum and minimum integers used by it. The $L(d,1)$–labelings–number of G is the minimum span over all $L(d,1)$-labelings of G.

This report focuses on $L(d,1)$–labeling–number of the edge–path–replacement of a graph.

Hadamard matrices, multipartite regular digraphs and imprimitive association schemes

Jianmin Ma

Hebei Normal University

I will talk about multipartite regular digraphs, tournaments, Hadamard matrices and small imprimitve association schemes.

These combinatorial configurations are mutually related.

A generalization of G–parking functions

Jun Ma

Department of Mathematics, Shanghai Jiao Tong University

The classical parking functions are defined as follows. There are n parking spaces which are arranged in a line, numbered 0 to $n-1$ left to right and n drivers labeled $1, \ldots, n$. Each driver i has an initial parking preference a_i. Drivers enter the parking area in the order in which they are labeled. Each driver proceeds to his preferred space and parks here if it is free, or parks at the next unoccupied space to the right. If all the drivers park successfully by this rule, then the sequence $(a_1, \ldots, a_n)$ is called a parking function.

Konheim and Weiss [1] introduced the conception of parking functions in the study of the linear probes of random hashing function. Riordan [3] studied a relation of parking problems to ballot problems. The most notable result about parking functions is a bijection from the set of classical parking functions of length n to the set of labeled trees on $n+1$ vertices.

Let G be a connected digraph with vertex set $V(G) = \{0, 1, 2, \ldots, n\}$ and edge set $E(G)$. Postnikov and Shapiro [2] introduced the conception of the G-parking functions, in the study of certain quotients of the polynomial ring. For the complete graph $G = K_{n+1}$ on $n+1$ vertices, K_{n+1}-parking functions are exactly the classical parking functions.

Let Δ be an integer $n \times n$–matrix which satisfy the conditions: $\det \Delta \neq 0$, $\Delta_{ij} \leq 0$ for $i \neq j$, and there exists a vector $r = (r_1, \cdots, r_n) > 0$ such that $r\Delta \geq 0$. Here the notation $r > 0$ means that $r_i > 0$ for all i and $r \geq r'$ means that $r_i \geq r'_i$ for every i. In this talk, we will introduce (Δ, r)–parking functions, which is a new generalization of the G–parking functions.

References:

[1] A.G. Konheim, B. Weiss, An occupancy discipline and applications, SIAM J. Appl. Math., 14 (1966), 1266-1274.

[2] A. Postnikov, B. Shapiro, Trees, Parking Functions, Syzygies, and Deformations of Monomial Ideals, Transactions of the American Mathematical Society, 356 (2004),3109-3142.

[3] J. Riordan, Ballots and trees, J. Combin. Theory, 6 (1969), 408-411.

Characterization and reduction of concept lattices through matroid theory

Hua Mao

Department of Mathematics, Hebei University, Baoding 071002, China

Concept lattice theory is an efficient tool for data analysis. Reduction of attributes and objects for a context is an important issue in concept lattice theory. We explore the relationships between matroidal spaces and concepts for a given context. With the assistance of matroid theory, we conduct the attribute reduction and object reduction for a context and formulate the concepts for a given context. As a generalization of the linear independence in vector spaces, matroids provide well established platforms for some algorithms such as greedy algorithm.

In this paper, we mainly propose a notion of matroidal space by the family of circuits of a matroid. These relationships characterize the constructions of not only concepts, but also concept lattices. Meanwhile, we demonstrate that reduction of attributes and objects proposed in this paper preserve the original hierarchy order in the concept lattice for a given context.

Characterization of $(2m, m)$–paintable graphs

Jixian Meng

Zhejiang Normal University

This paper proves that for any graph G and any positive integer m, G is $(2m, m)$–paintable if and only if G is 2–paintable. It was asked by Zhu in 2009 whether k–paintable graphs are (km, m)–paintable for any positive integer m. Our result answers this question in the affirmative for $k = 2$.

Directed short cycles and rainbow short cycles

Bo Ning

Department of Applied Mathematics, School of Science, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, P.R. China

The main topic of this talk is the existence of directed short cycles in oriented graphs and rainbow short cycles in edge–colored graphs. We will report new sufficient conditions for the existence of directed triangles in oriented graphs and the existence of directed quadrangles in oriented bipartite graphs. We will also report some new sufficient conditions for the existence of rainbow triangles in edge–colored graphs and rainbow quadrangles in edge–colored bipartite graphs. As corollaries, a conjecture of Li and Wang [3] and a conjecture of Li [1] are confirmed. (Joint work with Jun Ge [4], Binlong Li, Chuandong Xu and Shenggui Zhang [2])

Reference:

[1] H. Li, Rainbow C_3's and C_4's in edge-colored graphs, *Discrete Math.* **313** (19) (2013), 1893–1896.

[2] B. Li, B. Ning, C. Xu, S. Zhang, Rainbow triangles in edge-colored graphs, *European J. Combin.* **36** (2014), 453–459.

[3] H. Li, G. Wang, Color degree and heterochromatic cycles in edge-colored graphs, *European J. Combin.* **33** (8) (2012), 1958–1964.

[4] B. Ning, J. Ge, Rainbow C_4's and directed C_4's: the bipartite case study, *Bull. Malays. Math. Sci. Soc.* (2014+), to appear.

Paths and cycles in hypertournaments

Wenjie Ning

Department of Mathematical Sciences, Tsinghua University

A k–hypertournament H on n vertices with $2 \leq k \leq n$ is a pair $H = (V, A_H)$, where V is a set of n vertices and A_H is a set of k–tuples of vertices, called arcs, such that for any k-subset S of V, A_H contains exactly one of the $k!$ k–tuples whose entries belong to S. Obviously, a 2–hypertournament is a tournament.

Moon [J. W. Moon, On k-cyclic and pancyclic arcs in strong tournaments, *J. Combin. Inform. System Sci.* 19 (1994) 207-214] proved that for every strong tournament, there is a Hamiltonian cycle which contains at least three pancyclic arcs. In this talk, we will show that for an arbitrary strong k-hypertournament H with n vertices, where $2 \leq k \leq n-2$, there is a Hamiltonian cycle C containing at least three pancyclic arcs, each of which belongs to an m-cycle C_m for each $m \in \{3, 4, \ldots, n\}$ such that $V(C_3) \subset V(C_4) \subset \cdots \subset V(C_n)$.

The Kirchhoff indices and the matching numbers of unicyclic graphs

Xuli Qi

College of Mathematics and Information Science，Hebei Normal University，Shijiazhuang 050024, P. R. China

The Kirchhoff index of a connected graph is the sum of resistance distances between all unordered pairs of vertices in the graph. It found considerable applications in a variety of fields. In this paper, we determine the minimum Kirchhoff index among the unicyclic graphs with fixed number of vertices and matching number, and characterize the extremal graphs.

The least common multiple of consecutive quadratic progression terms

Guoyou Qian

Mathematical College, Sichuan University, Chengdu 610064, P.R. China

Let k be an arbitrary given positive integer and let $f(x) \in \mathbb{Z}[x]$ be a quadratic polynomial with D as its discriminant and a as the coefficient of its quadratic term. Associated to the least common multiple $\mathrm{lcm}_{0\le i\le k}\{f(n+i)\}$ of any $k+1$ consecutive terms in the quadratic progression $\{f(n)\}_{n\in\mathbb{N}^*}$, we define the function $g_{k,f}(n) := (\prod_{i=0}^{k}|f(n+i)|)/\mathrm{lcm}_{0\le i\le k}\{f(n+i)\}$ for all positive integers $n \in \mathbb{N}^* \setminus Z_{k,f}$, where $Z_{k,f} := \bigcup_{i=0}^{k}\{n \in \mathbb{N}^* : f(n+i) = 0\}$. Let $\mathcal{K}_f := \{j \in \mathbb{N}^* : D \neq a^2 i^2$ for all integers i with $1 \le i \le j\}$.

In this talk, we show that $g_{k,f}$ is eventually periodic if and only if $\mathcal{K}_f$ is nonempty and $k \in \mathcal{K}_f$. We introduce the concept of minimal distance among the roots of quadratic congruences and provide detailed p-adic analysis of $g_{k,f}$. By developing arithmetic properties of minimal distance, we determine the local periods, and then arrive at the determination of the exact value of the smallest period of $g_{k,f}$. We also obtain asymptotic formulae of $\log \mathrm{lcm}_{0\le i\le k}\{f(n+i)\}$ for all quadratic polynomials f as n goes to infinity.

A way finding $r(k, l)$ and $r(3, 10) = 41$

Xiurang Qiao

7#302 fifth area JiLi district LuoYang 471012 China

On basis of two definitions that 1. an induced subgraph by a vertex $v_i \in G$ and its neighbors in G is defined a vertex adjacent closed subgraph, and denoted by $Q_i(= G[V(Nv_i)])$, with the vertex v_i called the hub; 2. A graph on $r(k,l)-1$ vertices is called the (k,l)-Ramsey graph, denoted by $RG(k,l)$ if it contains only cliques of degree $k-1$,and the intersect $Q_i \cap Q_j$ of any two nonadjacent vertices i and j must contain only K_{k-2}; and its complement contains only cliques of degree $l-1$,and the intersect $Q_i \cap Q_j$ of any two nonadjacent vertices i and j must contain only K_{l-2}.

Two theorems that theorem 1: the biggest clique in G is contained in some Q_i of G, and theorem 2: $r(k,l) = \mid V(RG(k,l)) \mid +1$ are put forward and proved in this paper.

With those definitions and theorems as well as analysis of property of chords a method for quick inspection and building $RG(k,l)$ is proposed. Accordingly, $RG(10,3)$ and its complement are built, which are respectively the strongly 29–regular graph and the strongly 10–regular graph on orders 40. We have tested $RG(10,3)$ and its complement $RG(3,10)$, and gotten $r(3,10) = 41$.

Every contraction critical 8–connected graphs with minimum degree 9 is 9–regular

Chengfu Qin

School of Mathematics Science, Guangxi Teachers Education University, 530001, Nanning, P.R.China

Y. Egawa showed that any contraction critical k–connected graph has a fragment of cardinality at most $\frac{k}{4}$. Thus, for $4 \leq k \leq 7$, the minimum degree of contraction critically k–connected graphs is k. For a contraction critical 8–connected graph G, the minimum degree of G is either 8 or 9.

This paper we study the structure of contraction critical 8–connected graph with minimum degree 9. We show that G is contraction critical 8–connected graph with minimum degree 9 if and only if it can be get from contraction critical 4–connected graph by splitting each vertex into K_2.

Some combinatorial identities involing a new class of generalized λ–array type polynomials

Lan Qing[1], Wuyungaowa[2]

[1]School of Mathematics and Science, Inner Mongolia University

[2]Department of Mathematics, College of Sciences and Technology, Inner Mongolia University, Huhhot 010021, P. R. China

In this paper, we present some properties of new generalized array type polynomials and the generalized Hermite-based Apostol–Bernoulli polynomial containing parameters a, b, c , which involving generalization well-known number sequences or polynomials,and mean while we obtain some combinatorial identities between these new generalized sequences and thus get more related properties.we also give many applications involing this class of polynomials.

Cycle space and graph coloring

Han Ren, Weihua Lu, Chao Yang

East China Normal University

Many results have been obtained for graph colorings but seldom is known for these rooted at the cycle space. In this talk we study the relationship between cycle space and graph colorings. We shall show how the edge-coloring of a graph is affected by its cycle space.

莱布尼茨的中国情

沈灏

上海交通大学数学系

我们从以下四方面简要介绍莱布尼茨与中国传统文化的密切关系：

1. 莱布尼茨与来华耶稣会传教士；
2. 中国文化的热情传播者，中西交流的有力推动者；
3. 为了照亮我们这个时代 –《中国近事》的编辑与出版；
4. 二进制与《易经》.

$(2,1)$–Total labeling of trees

Qiaojun Shu

Hangzhou Dianzi University, Hangzhou, 310018, China

A k–(2,1)–total labeling of a graph G is to label the vertices and the edges of G using integers from 0 to k such that all adjacent vertices as well as edges receive different labels, and the difference between the labels of a vertex and its incident edges is at least 2. The $(2,1)$–total labeling number $\lambda_2^t(G)$ is the smallest integer k such that G has a k–$(2,1)$–total labeling. It is known that every tree T with maximum degree Δ has $\Delta + 1 \leq \lambda_2^t(T) \leq \Delta + 2$.

In this talk, we provide several sufficient conditions for a tree T to have $\lambda_2^t(T) = \Delta + 1$ when $\Delta \geq 4$.

On r–hued coloring of planar graphs with girth at least 6

Huimin Song[1], Hong-Jian Lai[2], Jianliang[3] Wu

[1]School of Mathematics and Statistics, Shandong University，Weihai 264209,P.R.China

[2]Department of Mathematics, West Virginia University, Morgantown, WV 26506-6310, USA

[3]Department of Mathematics, Shandong University, Jinan 250010, P.R.China

For integers $k, r > 0$, a (k, r)–coloring of a graph G is a proper coloring c such that for any vertex v with degree $d(v)$, v is adjacent to at least $\min\{d(v), r\}$ different colors. Such coloring is also called as an r-hued coloring. The *r–hued chromatic number* of G, $\chi_r(G)$, is the least integer k such that a (k, r)–coloring of G exists.

In this paper, we proved that if G is a planar graph with girth at least 6, then $\chi_r(G) \leq r + 5$. This extends a former result in [Comb. Optim. 24 (2012) 580-592]. It also implies that a conjecture on r-hued coloring of planar graphs is true for planar graphs with girth at least 6.

Tilings of right trapezoids with similar triangles

Zhanjun Su

College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang, 050024, China

We say that a triangle $\triangle$ tiles the polygon $\mathcal{P}$, if $\mathcal{P}$ can be decomposed into finitely many non-overlapping triangles similar to $\triangle$. A tiling is called regular if there are two angles of the triangles, say α and β, such that at each vertex V of the tiling the number of triangles having V as a vertex and having angle α at V is the same as the number of triangles having angle β at V. Otherwise the tiling is called irregular. Denote by $\mathcal{R}(\delta)$ the right trapezoid with acute angle δ. In this paper we first show that any right triangle can not regularly tile $\mathcal{R}(\delta)$ and prove that if a right triangle $T = (\alpha, \beta, \pi/2)$ irregularly tiles $\mathcal{R}(\delta)$ then $(\alpha, \beta) = (\delta, \pi/2 - \delta)$ or $(\alpha, \beta) = (\delta/2, \pi/2 - \delta/2)$. Secondly, We show that if a non-right triangle T tiles $\mathcal{R}(\delta)$ then the angles of T are given by one of the four triples $(\pi/6, \pi/6, 2\pi/3)$, $(\pi/8, \pi/4, 5\pi/8)$, $(\pi/4, \pi/3, 5\pi/12)$, and $(\pi/12, \pi/4, 2\pi/3)$.

4–Factor–criticality of vertex–transitive graphs

Wuyang Sun[1,2], Heping Zhang[1]

[1]School of Mathematics and Statistics, Lanzhou University

[2]Center of Discrete Mathematics and Theoretical Computer Science, Fuzhou University

A graph of order n is *p–factor–critical*, where p is an integer of the same parity as n, if the removal of any set of p vertices results in a graph with a perfect matching. 1–factor–critical graphs and 2–factor–critical graphs are well–known factor–critical graphs and bicritical graphs, respectively. It is known that if a connected vertex-transitive graph has odd order, then it is factor–critical, otherwise it is elementary bipartite or bicritical.

In this paper, we show that a connected vertex–transitive non–bipartite graph of even order at least 6 is 4–factor–critical if and only if its degree is at least 5. This result implies that each connected non-bipartite Cayley graphs of even order and degree at least 5 is 2–extendable.

Generalized 3–edge–connectivity of Cartesian product graphs

Yuefang Sun[1], Sanming Zhou[2]

[1] Department of Mathematics, Shaoxing University, Zhejiang 312000, P. R. China

[2] Department of Mathematics and Statistics,The University of Melbourne, Parkville, VIC 3010, Australia

The generalized k–connectivity $\kappa_k(G)$ of a graph G was introduced by Chartrand et al. in 1984. As a natural counterpart of this concept, Li et al. in 2011 introduced the concept of generalized k–edge–connectivity which is defined as $\lambda_k(G) = \min\{\lambda(S) : S \subseteq V(G) \text{ and } |S| = k\}$, where $\lambda(S)$ denote the maximum number ℓ of pairwise edge–disjoint trees $T_1, T_2, \ldots, T_\ell$ in G such that $S \subseteq V(T_i)$ for $1 \leq i \leq \ell$.

In this paper we prove that for any two connected graphs G and H, we have $\lambda_3(G\square H) \geq \lambda_3(G) + \lambda_3(H)$, where $G\square H$ is the Cartesian product of G and H. Moreover, the bound is sharp. We also obtain the precise values for the generalized 3–edge–connectivity of the Cartesian product of some special graph classes.

Bounding k–separated matchings and independent sets in sparse random graphs and partial (n, k, ℓ, λ)–hypergraphs

Fang Tian[1], Zilong Liu[2]

[1]Department of Applied Mathematics, Shanghai University of Finance and Economics, Shanghai, 200433, China

[2]School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai, 200093, China

The lower and upper bounds of the random variable, the size of the maximum k-separated matching for any positive integer $k \geq 3$, denoted by $um_k(G_{n,p})$, are considered in sparse random graph $G_{n,p}$ with $p = p(n) = c/n$ for some sufficiently large constant c, where the lower bound

$$um_k(G_{n,p}) > \frac{n}{4c^{k-1}}\left[(k-1-\varepsilon)\log c - \log\log c - \log(k-1-\varepsilon) + 2 + 2\log 2\right]$$

generalizes the results of R. J. Kang and P. Manggala.

For given integers $1 < \ell < k$, a k-graph $\mathcal{H}$ is called a partial design with parameters (n, k, ℓ, λ) if the codegree of every ℓ-subset of $V(\mathcal{H})$ is at most some positive integer λ. It is also called a partial Steiner (n, k, ℓ)-system as $\lambda = 1$. Define $f(n, k, \ell, \lambda) = \min\{\alpha(\mathcal{H}) |\ \mathcal{H} \text{ is a partial } (n, k, \ell, \lambda) - \text{design}\}$. We show $f(n, k, \ell, \lambda) \geq$

$\mathcal{C}\left(\frac{n}{\lambda}\log\frac{n}{\lambda}\right)^{\frac{1}{\ell}}$ as $k \geq 5$, $\lceil\frac{2k+4}{5}\rceil \leq \ell \leq k-2$ and $\lambda \leq \left\lfloor\frac{n^{\frac{5\ell-2k-4}{3k-9}}}{\omega(n)}\right\rfloor$, where $\mathcal{C}$ is any constant less than $\left(\frac{[3\ell(k-\ell)-3k+4](\ell-1)!}{-2^{k-1}(3k-4)(k-\ell)\log(1-2^{-k+1})}\right)^{\frac{1}{\ell}}$ and $\omega(n)$ is any function tending arbitrarily slowly to infinity with n, extending the aforementioned results of Kostochka, Mubayi, and Verstraete for the cases $\ell = k-1$ or $\lambda = 1$. An upper bound $f(n,k,\ell,\lambda) \lesssim (\frac{k^\ell}{\lambda})^{\frac{1}{k-1}} n^{\frac{k-\ell}{k-1}}\left(\log\frac{n}{\lambda}\right)^{\frac{1}{k-1}}$ holds for $\log n = o(\lambda)$ and $\log\lambda = o(\log n)$ by constructing random hypergraphs, where $\log x$ is the natural logarithmic function.

If time perimitted, we will discuss how to use stochastic process to construct hypergraph matching algorithms with applications in partial Steiner (n,k,ℓ)-systems. They are joint works with Jiaming Zhang, Zilong Liu.

The Q–generating function for graphs with application

Guixian Tian

Zhejiang Normal University

For a simple connected graph G, the Q–generating function of the numbers N_k of semi-edge walks of length k in G is defined by $W_Q(t) = \sum_{k=0}^{\infty} N_k t^k$. This paper reveals that the Q–generating function $W_Q(t)$ may be expressed in terms of the Q–polynomials of the graph G and its complement $\overline{G}$. Using this result, we study some Q–spectral properties of graphs and compute the Q–polynomials for some graphs obtained by the use of some operation on graphs, such as the complement graph of a regular graph, the join of two graphs, the (edge)corona of two graphs and so forth. As another application of the Q-generating function $W_Q(t)$, we also give a combinatorial interpretation of the Q-coronal of G, which is defined to be the sum of the entries of the matrix $(\lambda I_n - Q(G))^{-1}$. This result may be used to obtain the many alternative calculations of the Q–polynomials of the (edge)corona of two graphs. Further, we also compute the Q-coronals of the join of two graphs and the complete multipartite graphs.

The work is joint with Shuyu Cui (ZJNU).

On 2–factors in claw–free graphs whose edges are in small cycles

Runli Tian, Liming Xiong

Department of Mathematics, Beijing Institute of Technology, Beijing 100081, China

It is showed that every simple claw–free graph of minimum degree at least 3 in which every edge lies in a cycle of length at most 5 has a 2–factor.

Equitable total coloring of generalized Petersen graphs $P(n, k)$

Chunling Tong[1], Xiaohui Lin[2], Yuansheng Yang[2], Aijun Dong[3]

[1]College of Information Science and Electricity Engineering, Shandong Jiaotong University, Jinan 250023, China

[2]College of Computer Science, Dalian University of Technology, Dalian 116024, China

[3] College of Science, Shandong Jiaotong University, Jinan 250023, China

A k–*total colouring* of a graph G is a map σ: $V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ such that no two adjacent or incident elements of $V(G) \cup E(G)$ receive the same colour. A k-total colouring is *equitable* if $||\sigma^{-1}(i)| - |\sigma^{-1}(j)|| \leq 1$ for each pair of distinct colours i and j $(1 \leq i, j \leq k)$. The *equitable total chromatic number* is the smallest one out of all the k for which G has an equitable total k-colouring.

In this paper, we show that the equitable total chromatic number of $P(n, k)$ is 4 for all $k \not\equiv 0 \pmod 4$ except $P(5, 1)$ and $P(9, 3)$. The result determines the exact value of $\chi''(P(n, k))$, also is relevant as an evidence that every cubic graph is such that the total chromatic number is equal to the equitable total chromatic number.

The k–path vertex cover problem

Jianhua Tu

Beijing University of Chemical Technology

Given a graph $G = (V, E)$, a subset of vertices $S \subset V$ is a *k-path vertex cover* if every path of order k in G contains at least one vertex from S. *The k–path vertex cover problem*(VCP_k) is to find a minimum cardinality k-path vertex cover. The k–path vertex cover number, $\psi(G)$, of G is the cardinality of a minimum k-path vertex cover for G. Clearly, the vertex cover problem is VCP_2, VCP_k generalizes the well-known vertex cover problem. VCP_k is also a special case of *the vertex deletion problem*: find a minimum subset of vertices whose deletion gives a graph satisfying a given property. VCP_k is to find a set $S \subseteq V$ of minimum size such that $G[V - S]$ contains no P_k, where P_k is a path with k vertices.

In this talk, we investigate VCP_k from three aspects: (1) The k–path vertex cover number, $\psi(G)$; (2) Exact algorithm and approximation algorithm; (3) Parameter complexity.

The log–concavity of genus polynomials

David G.L. Wang

School of Mathematics and Statistics, Beijing Institute of Technology, 102488 Beijing, P.R. China

Motivated by the 25 years old conjecture that the genus polynomial of every graph is log–concave, we formulate conditions on a set of log–concave sequences, under which any linear combination of those sequences is log-concave. In particular, we show that the 4–wheel–linear graphs have log–concave genus polynomials.

Algebraic characterizations of graphs with distinct eigenvalues

Jianfeng Wang[1,2]

[1]Center for Combinatorics and LPMC-TJKLC, Nankai University 300071, Tianjin, China

[2]Department of Mathematics, Qinghai Normal University, Xining, Qinghai 810008, China

We first investigate the Radić matrix with distinct eigenvalues, which inspires us to study the more general case, i.e., Hermitian matrices. As a result, we obtain an algebraic characterization to a graph with k distinct eigenvalues with respect to the adjacency and (normalized) Laplacian matrix.

This is a joint work with Professors Xueliang Li and Qiongxiang Huang.

Some formulas related to residue method

Sudan Wang

Inner Mongolia University

In this thesis, with the help of residue method we find some interesting formulas relate residue and ordinary Bell polynomials $\hat{B}_{n,k}(x_1, x_2, \cdots)$. Further, we prove identities involving some combinatorial numbers to demonstrate the application of the results above.

有关 Bernoulli 多项式与 Euler 多项式的混合乘积之和的研究

王伟平

浙江理工大学理学院

By the methods of partial fraction decomposition and generating function, we study the mixed sums of products of l Bernoulli polynomials and $k-l$ Euler polynomials, which are of the form

$$T_{n,k}^{\lambda}(y;l,k-l) := \sum_{\substack{j_1+\cdots+j_k=n\\ j_1,\ldots,j_k\geq 0}} \prod_{i=1}^{k} \lambda_i^{j_i} \binom{n}{j_1,\ldots,j_k} \prod_{p=1}^{l} B_{j_p}(x_p) \prod_{q=l+1}^{k} E_{j_q}(x_q)\,,$$

where $\lambda=(\lambda_1,\ldots,\lambda_k)$, and $\lambda_1,\ldots,\lambda_k$ are nonzero rational numbers.

Finding the least element of the ordering of graphs with respect to their matching numbers

Wasin So[1], Wenhuan Wang[2]

[1]Department of Mathematics, San Jose State University, San Jose, California 95192-0103, USA

[2]Department of Mathematics, Shanghai University, Shanghai 200444, China

Given two graphs G_1 and G_2, the ordering $G_1 \preccurlyeq G_2$ is defined by $m(G_1,k)\leq m(G_2,k)$ for all k, where $m(G_i,k)$ is the number of k–matchings in G_i. The existence of the least element of $\preccurlyeq$ among connected graphs of order n and size m is conjectured. Special cases of $m=n-1,n$, and $n+1$ are known in literature. In this paper, we further affirm the conjecture by finding the least elements for the cases of $n-1\leq m\leq 2n-3$ and $\frac{n(n-1)}{2}-(n-2)\leq m\leq \frac{n(n-1)}{2}$. As consequences, we determine the graphs with the minimum matching energy, and with the minimum Hosoya index among connected graphs of order n and size m where $n-1\leq m\leq 2n-3$ and $\frac{n(n-1)}{2}-(n-2)\leq m\leq \frac{n(n-1)}{2}$.

Whipple–like formulas for terminating ${}_3F_2$–series

Xiaoxia Wang

Shanghai University, Department of Mathematics，Shanghai 200444, P. R. China

Gould–Hsu inversions are utilized to investigate the dual relations of Dougall's formula for the very well–poised ${}_5F_4$–series. Several identities are established for terminating hypergeometric ${}_3F_2$–series, which resemble both Watson and Whipple formulae in parameter structure.

Log–behavior of combinatorial sequences

Xingwei Wang

Nankai University

We introduce the notion of infinitely log–monotonic sequences. We show that the sequences of the Bernoulli numbers, the Catalan numbers and the central binomial coefficients are infinitely log–monotonic. In particular, if a sequence $\{a_n\}_{n\geq 0}$ is log-monotonic of order two, then it is ratio log–concave in the sense that the sequence $\{a_{n+1}/a_n\}_{n\geq 0}$ is log–concave. Furthermore, we prove that if a sequence $\{a_n\}_{n\geq k}$ is ratio log–concave, then the sequence $\{\sqrt[n]{a_n}\}_{n\geq k}$ is strictly log–concave subject to a certain initial condition. As consequences, we show that the sequences of the derangement numbers, the Motzkin numbers, the Fine numbers, the central Delannoy numbers, the numbers of treelike polyhexes and the Domb numbers are ratio log–concave. For the case of the Domb numbers D_n, we confirm a conjecture of Sun on the log–concavity of the sequence $\{\sqrt[n]{D_n}\}_{n\geq 1}$. For the partition function $p(n)$, we prove the conjecture of Desalvo and Pak that for $n \geq 45$ $\frac{p(n-1)}{p(n)}\left(1+\frac{\pi}{\sqrt{24}n^{3/2}}\right) > \frac{p(n)}{p(n+1)}$. pWe also show that for given $r \geq 1$ and sufficiently large n, $(-1)^{r-1}\Delta^r \log p(n) > 0$.

Linear 2–arboricity of planar graphs

Yiqiao Wang

School of Management, Beijing University of Chinese Medicine, Beijing 100029, China

The linear 2–arboricity $\mathrm{la}_2(G)$ of a graph G is the least integer k such that G can be partitioned into k edge–disjoint forests, whose component trees are paths of length at most 2.

In this talk, we study the linear 2–arboricity of planar graphs by showing the following results: (1) For any planar graph G, $\mathrm{la}_2(G) \leq \lceil(\Delta+1)/2\rceil + 6$; (2) If a planar graph G contains no 3–cycle sharing exactly one edge with a 4–cycle, then $\mathrm{la}_2(G) \leq \lceil(\Delta+1)/2\rceil + 4$. Here Δ denotes the maximum degree of a vertex in the graph G.

Evaluations of q–Watson type, q–Dixon type, q–Whipple type and q–Gosper type series

Chuanan Wei

Hainan Medical University

According to the method of series rearrangement, several extensions of q–Watson formulas with two extra integer parameters are established. Then they and sear's transformation formula are used to derive some extensions of q–Dixon formulas and q–Whipple formulas with two extra integer parameters. Meanwhile, the generalizations of two q–Gosper formulas with one additional integer parameter are also given in the same way. As the special cases of these results, many interesting evaluations of q-Watson type, q–Dixon type, q–Whipple type and q–Gosper type series are displayed.

化学指数的一些进展

魏福义

华南农业大学数学系

在此报告中, 我们将介绍一些在化学指数研究方向上所取得的新的研究结果.

One problem and three conjectures on generalized Fibonacci cubes

Jianxin Wei

Ludong University

A binary word f is a factor of word u if f appears as a sequence of $|f|$ consecutive bits of u, where $|f|$ denotes the length of f. Generalized Fibonacci cube $Q_d(f)$ is the graph obtained from the d-hypercube Q_d by removing all vertices that contain f as a factor. This concept was posed by Ilić, Klavžar and Rho. If $Q_d(f)$ is an isometric subgraph of Q_d for all $d \geq 1$, then f is called good, otherwise it is bad. The index of a binary string f, denoted by $B(f)$, is the smallest integer d such that $Q_d(f)$ is not an isometric subgraph of Q_d. Ilić et al. posed the following problem and conjectures:

Problem 1. *Suppose $Q_d(f)$ is not an isometric subgraph Q_d. Is there a dimension d' such that $Q_d(f)$ can be isometrically embedded into $Q_{d'}$?*

Conjecture 2. *For any bad word f, $B(f) < 2|f|$.*

Conjecture 3. *If $Q_d(f)$ is an isometric subgraph of Q_d, then $Q_d(ff)$ is an isometric subgraph of Q_d.*

A binary word u is called f-free if it contains no f as a factor. The word f is d-good if for any f-free words u and v of length d, v can be obtained from u by complementing one by one the bits of u on which u and v differ, such that all intermediate words are f-free. Such a process is called an f-free transformation of u to v. The word f is s-isometric if for any binary words u and v of the same length that differ in s bits there is an f-free transformation of u to v. Ilić et al. posed another conjecture:

Conjecture 4. *Let f be a bad word that contains exactly two 1s. Then f is 2-isometric if and only if $f = 1^{2^r-1}01^{2^r-1}01^{2^{(r+1)}}$ (or its reverse) for some $r \geq 0$.*

We give a negative answer to Problem 1, confirm Conjectures 2 and 3 and show that Conjectures 4 is not true by the following theorems:

Theorem 1. *Suppose $Q_d(f)$ is not an isometric subgraph Q_d. There exists no d' such that $Q_d(f)$ can be isometrically embedded into $Q_{d'}$.*

Theorem 2. *If f is a bad word, then $B(f) < 2|f|$.*

Theorem 3. *If $Q_d(f)$ is an isometric subgraph of Q_d, then $Q_d(ff)$ is an isometric subgraph of Q_d.*

Theorem 4. *Let f be a bad word that contains exactly two 1s. Then f is 2-isometric if and only if $f = 1^r01^r01^{2(r+1)}$ (or its reverse) for some $r \geq 0$.*

The surviving rate of planar graphs

Tingting Wu

Zhejiang Normal University, Jinhua

Let G be a connected graph with $n \geq 2$ vertices. Let $k \geq 1$ be an integer. Suppose that a fire breaks out at a vertex v of G. A firefighter starts to protect vertices. At each step, the firefighter protects k-vertices not yet on fire. At the end of each step, the fire spreads to all the unprotected vertices that have a neighbour on fire. Let $\mathrm{sn}_k(v)$ denote the maximum number of vertices in G that the firefighter can save when a fire breaks out at vertex v. The k-surviving rate $\rho_k(G)$ of G is defined to be $\frac{1}{n^2}\sum_{v\in V(G)} \mathrm{sn}_k(v)$, which is the average proportion of saved vertices. Given a class of graphs $\mathcal{G}$, we say that $\mathcal{G}$ is *k-good* if $\rho_k(G) \geq c > 0$ for all $G \in \mathcal{G}$, and $\mathcal{G}$ is *k-optimal* if $\lim\limits_{n\to\infty} \rho_k(G) = 1$ for $G \in \mathcal{G}$.

In this talk, we give a brief survey on the recent progress of this subject and show that some planar graphs, e.g., planar graphs without 5-cycle having a chord, are 2-good.

On the Merrifield–Simmons index and Hosoya index of tree–triangle graphs

Sun Xie[1], Haixing Zhao[1], Yaping Mao[2]

[1]School of Computer, Qinghai Normal, University, Xining, Qinghai 810008, China

[2]Department of Mathematics, Qinghai Normal, University, Xining, Qinghai 810008, China

The Hosoya index and the Merrifield–Simmons index are typical examples of graph invariants used in mathematical chemistry for quantifying relevant details of molecular structure. Denote by $Z(G)$ and $\sigma(G)$ the Hosoya index and the Merrifield–Simmons index of a graph G. Let G_k be a connected graph with k triangles such that each pair of triangles has at most one common vertex.

In this paper, Denote by $\mathcal{G}(k,\ell)$ the set of all tree–triangle graphs with k triangle and diameter ℓ. In this paper, the maximal and the minimal Hosoya index and Merrifield–Simmons index for a graph G_k in $\mathcal{G}(k,\ell)$ are determined. In the end, we give an algorithm to count the number of the Hosoya indices and Merrifield–Simmons indices of G_k.

Graphs characterized by the second distance eigenvalue

Rundan Xing

School of Computer Science, Wuyi University

In this talk, we consider the second largest eigenvalue of the distance matrix of graphs. We characterize all connected graphs with second distance eigenvalue belonging to $(-\infty, -0.5858)$, and all trees and unicyclic graphs whose second distance eigenvalues belong to $(-\infty, -0.5)$ with the exception of a class of unicyclic graphs. We also investigate the distance spectral gaps of trees and unicyclic graphs.

Balance domination in graphs

Baogen Xu

Department of Mathematics, East China Jiaotong University, Nanchang,Jiangxi 330013,P.R.China

Let $G = (V, E)$ be a graph. A function $f : V \to \{-1, 0, +1\}$ is said to be a balance dominating function (BDF) of G if $\sum\limits_{u \in N[v]} f(u) = 0$ holds for every vertex $v \in V(G)$, the balance domination number $\gamma_b(G)$ of G is defined as

$$\gamma_b(G) = \max\left\{\sum_{v \in V} f(v) | f \text{ is a } BDF \text{ of } G\right\}.$$

In this paper we introduce the above concept for balance domination in graphs, obtain some bounds for the balance domination number, and determine the exact value of $\gamma_b(G)$ for some special classes of graphs. In addition, we pose some open problems and conjectures.

Complete forcing numbers of graphs

Shoujun Xu, Heping Zhang

School of Mathematics and Statistics, Gansu Key Laboratory of Applied Mathematics and Complex Systems, Lanzhou University, Lanzhou, Gansu 730000, China

Let G be a graph with edge set $E(G)$ that admits a perfect matching M. A *forcing set* of M is a subset of M contained in no other perfect matchings of G. A *complete forcing set* of G, recently introduced by Xu et al. [Complete forcing numbers of catacondensed hexagonal systems, *J. Comb. Optim.*, doi: 10.1007/s10878-013-9624-x], is a subset of $E(G)$ on which the restriction of any perfect matching M is a forcing set of M. The minimum possible cardinality of complete forcing sets of G is the *complete forcing number* of G.

In this talk, we will give the motivation of introducing these comcepts, and survey some recent results, mainly including the sufficient and necessary conditions for the complete forcing sets of graphs, an linear algorithm for the complete forcing number of catacondensed benzenoid systems, explicit formulae for the complete forcing numbers of some typical graphs, such as benzenoid chains, coronoid chains, rectangular grids, triangular benzenoid systems, etc.

Comparison theorems on resistance distances and Kirchhoff indices of $S, T-$isomers

Yujun Yang

School of Mathematics and Information Sciences, Yantai University

Comparison theorems on resistance distances and Kirchhoff indices of the so-called $S-$ & $T-$isomer graphs are established. Then these results are applied to compare Kirchhoff indices of hexagonal chains, showing that the straight chain is the unique chain with maximum Kirchhoff index, whereas the minimum Kirchhoff index is achieved only when the hexagonal chain is an "all-kink" chain.

Some results of neighbor sum distinguishing colorings of graphs

Jingjing Yao

School of Science, Hebei University of Technology, Tianjin 300401, P.R.China

Let $G = (V,E)$ be a graph. A proper k-total coloring of graph G is a mapping from $V(G) \cup E(G)$ to $\{1, 2, \ldots, k\}$ such no two adjacent or incident elements in $V(G) \cup E(G)$ receive the same color. Let $f(v)$ denote the sum of the color on vertex v and the colors on the edges incident with v. A proper k-total coloring of graph G is called a neighbor sum distinguishing k-total coloring if $f(u) \neq f(v)$ for each edge $uv \in E(G)$. Similarly, we can define neighbor sum distinguishing edge coloring of graph G.

In this talk, we will give some results of those two coloring problems.

On the spectral radius of trees given diameter and edge independence number

Yanhong Yao

School of Mathematics and Statistics, Anyang Normal University, Anyang, 455000, P.R. China

This paper obtain the sharp upper bounds of spectral radius of trees given diameter and edge independence number, and give all trees whose spectral radius reach the supper bounds.

Some results and open problems on the primitive degree of nonnegative tensors

Pingzhi Yuan, Zilong He, Lihua You

School of Mathematical Sciences, South China Normal University, Guangzhou, 510631, P.R. China

Let $\mathbb{A}$ be a nonnegative primitive tensor with order m and dimension n. In this talk, we show that its primitive degree $\gamma(\mathbb{A}) \leq (n-1)^2 + 1$, and the upper bound is sharp. This confirms a conjecture of Shao (A general product of tensors with applications, Linear Algebra and its Appl. 439(2013), 2350-2366). We also show that the exponent set of nonnegative primitive tensors with order $m(\geq 3)$ and dimension n is $\{1, 2, \ldots, (n-1)^2 + 1\}$, which implies that the results of the case $m \geq 3$ (the case of tensors) is totally different from the case $m = 2$ (the case of matrices). Further, we propose some open problems for further research.

Some results on extreme theory of graph spectra

Guanglong Yu

Department of Mathematics, Yancheng Teachers University, Yancheng, 224002, Jiangsu, China

Recall that the central problem of the classical extremal graph theory is of the following type:

Problem A: Given a graph F, what is the maximum number of edges of a graph of order n, with no subgraph isomorphic to F?

Problem B: Given a class of graphs, what is the maximum number of edges of a graph of order n in this class?

During the past decades, subtler versions of Problems A, B have been investigated in the theory of graph spectra extensively. This talk will start with an simple introduction of these results. Most of the talk, however, will be dedicated to very recent results.

Neighbor sum distinguishing edge coloring of graphs

Xiaowei Yu

School of Mathematics, Shandong University, Jinan 250100,China

A *proper edge–k–coloring* of a graph G is a mapping from $E(G)$ to $\{1, 2, \ldots, k\}$ such that no two adjacent edges receive the same color. A proper edge–k–coloring of G

is called *neighbor sum distinguishing* if for each edge $uv \in E(G)$, the sum of colors taken on the edges incident to u is different from the sum of colors taken on the edges incident to v. Let $\chi'_{\sum}(G)$ denote the smallest value k in such a coloring of G. The definition of this parameter, which makes sense for graphs containing no isolated edges (we call such a graph *normal*). The *average degree* $\mathrm{ad}(G)$ of G denote by $\mathrm{ad}(G) = \frac{2|E(G)|}{|V(G)|}$. The *maximum average degree* $\mathrm{mad}(G)$ of G is the maximum of the average degree of its non–empty subgraphs.

It was conjectured by Flandrin et al. that $\chi'_{\sum}(G) \leq \Delta(G)+2$ for all normal graphs, except for C_5. It is proved that this conjecture holds for graphs with bounded maximum average degree. Our approach is based on the Combinatorial Nullstellensatz and the discharging method. We will introduce these results and the sketch of the proof.

无理的对角有序的幻方的存在性

余黄生

广西师范大学数学与统计学院

记 $M = nA+B$, 其中 $A = (a_{i,j})$, $B = (b_{i,j})$, 并且 $m_{i,j} = na_{i,j}+b_{i,j}$, $0 \leq a_{i,j}$, $b_{i,j} \leq n-1$. 一个 n 阶矩阵 $M = (m_{i,j})$ 称为 n 阶无理的对角有序的幻方, 若 M 满足:

(1) 每行 (和列) 和每条对角线上的 n 个数之和相同;

(2) 对所有 $0 \leq i \leq n-2$, 有 $m_{i,i} < m_{i+1,i+1}$, 和 $m_{n-1-i,i} < m_{n-2-i,i+1}$;

(3) A 和 B 中至少有一行 (或列) 上的 n 个数之和与其它行 (或列) 上的 n 个数之和不相同.

在本文中, 我们利用加框构造的方法, 证明了对所有 $n \geq 5$, 存在 n 阶无理的对角有序的幻方.

Unsplittable sequences over cyclic groups

Pingzhi Yuan

School of Mathematical Sciences, South China Normal University,
Guangzhou, 510631, P.R. China

Let G be a finite cyclic group of order $n \geq 2$. Every sequence S over G can be written in the form $S = (n_1 g) \cdot \ldots \cdot (n_l g)$ where $g \in G$ and $n_1, \ldots, n_l \in [1, ord(g)]$, and the index $ind(S)$ of S is defined as the minimum of $(n_1 + \cdots + n_l)/ord(g)$ over all $g \in G$ with $ord(g) = n$. In this talk, it is shown that any sequence S over G of length $|S| \geq n \geq 5$, $2 \not| n$ having an element with multiplicity at least $\frac{n}{3}$ has a subsequence T with $ind(T) = 1$. On the other hand, if $n, d \geq 2$ are positive integers with $d|n$ and

$n > d^2(d^3 - d^2 + d + 1)$, we provide an example of a sequence S of length $|S| \geq n$ having an element with multiplicity $l = \frac{n}{d} - d(d-1) - 1$ such that S has no subsequence T with $ind(T) = 1$, giving a general counterexample to a conjecture of Lemke and Kleitman.

S is called unsplittable if there do not exist an element g in G and two elements x, y in G such that $g = x + y$ and the new sequence $Sg^{-1}xy$ is still a minimal zero-sum sequence. In this talk we also talk about the structures of long unsplittable minimal zero-sun sequences. Our main result characterizes the structures of all such sequences S and shows that the index of S is at most 2, provided that the length of S is greater than or equal to $\lfloor \frac{n}{3} \rfloor + 8$ where $n > 20585$ is a positive integer with least prime divisor greater than 13.

Maxima of the Q-index of graph G with forbidden subgraph

Xiying Yuan

Department of Mathematics, Shanghai University, Shanghai, 200444, China

Let $q(G)$ be the Q-index (the largest eigenvalue of the signless Laplacian) of G.

Given a graph F, what is the maximum Q-index of a graph $G \in \mathcal{G}(n)$ with forbidden subgraph isomorphic to F?

In this talk we will consider the cases when F is a path or a cycle. Let $S_{n,k}$ be the graph obtained by joining each vertex of a complete graph of order k to each vertex of an independent set of order $n-k$, and let $S_{n,k}^{+}$ be the graph obtained by adding an edge to $S_{n,k}$. The following results will be introduced.

1. (V. Nikiforov, X. Yuan*, 2013) Let $k \geq 1$, $n \geq 7k^2$, and let G be a graph of order n.

   (i) if $q(G) \geq q(S_{n,k})$, then $P_{2k+2} \subset G$, unless $G = S_{n,k}$;

   (ii) if $q(G) \geq q\left(S_{n,k}^{+}\right)$, then $P_{2k+3} \subset G$, unless $G = S_{n,k}^{+}$.

2. (X. Yuan, 2014) Let $k \geq 3$, $n \geq 110k^2$, and G be a graph of order n. if $q(G) \geq q(S_{n,k})$, then $C_{2k+1} \subset G$, unless $G = S_{n,k}$.

3. (V. Nikiforov, X. Yuan*, 2014+) Let $k \geq 2$, $n \geq 400k^2$, and G be a graph of order n. if $q(G) \geq q\left(S_{n,k}^{+}\right)$, then $C_{2k+2} \subset G$, unless $G = S_{n,k}^{+}$.

The results about the cycle prove the conjecture in [M.A.A. de Freitas, V. Nikiforov, and L. Patuzzi, Maxima of the Q-index: forbidden 4-cycle and 5-cycle, *Electron. J. Linear Algebra* 26 (2013), 905-916.]

Part of the work joint with Prof. Vladimir Nikiforov, Department of Mathematical Sciences, University of Memphis, Memphis TN 38152, USA

Right triple convexity

Liping Yuan

College of Mathematics and Information Science Hebei Normal University

A set M in $\mathcal{R}$ is rt-convex if every pair of its points is included in a 3-point subset $\{x, y, z\}$ of M satisfying $\angle xyz = \pi/2$. We characterize rt-convex sets, and investigate rt-convexity for 2-connected polygonally connected sets, for 3-connected sets, for geometric graphs, for starshaped sets, and for finite sets.

最优组合批处理码的单调性质及上下界

张更生

河北师范大学数学与信息科学学院, 石家庄 050024

具有参数 n,k,m 的组合批处理码可以看做是一个 n 元集以及它的 m 个子集组成的集合系统, 满足对于任意 k 个元素都能经由从每个子集中至多取一 (可以一般化为 t) 个元素来取得. 在此系统中, 一个优化问题是确定 m 个子集中元素总数的最小值 $N(n,k,m)$. 这一问题不仅具有理论意义, 而且在数据的存储与恢复等应用领域有着重要价值.

本文研究了当参数 n,k,m 变化时 $N(n,k,m)$ 的变化规律; 给出了 $N(n,k,m)$ 的一个上下界：当 $2 \le k < m \le n-3$ 时, 如果 $m+1-k \ge \lceil\sqrt{k+1}\rceil$,

$$(n-m)k+m \ge N(n,k,m) \ge 2n-m+k-6+\left\lceil 2\sqrt{k+1}\right\rceil;$$

如果 $m+1-k < \lceil\sqrt{k+1}\rceil$,

$$(n-m)k+m \ge N(n,k,m) \ge 2n-6+\left\lceil 1+\frac{k+1}{m-k+1}\right\rceil;$$

确定了 $N(m+3,4,m)=m+9$(当 $m \ge 6$ 时), $N(8,4,5)=15$. 得到的结果部分解决了 Paterson 等人提出的未解决问题.

On the minimal energy of graphs

Jianbin Zhang

School of Mathematics, South China Normal University, Guangzhou, 510631, China

The energy of a graph is the sum of the absolute values of the eigenvalues of the graph which is used to approximate the total π-electron energy of the molecule.

In this paper, we determine the (n,e)-graphs with minimal energy for $e=n+1$ and $n+2$, which giving a complete solution to the conjecture for $e=n+1$ and $e=n+2$ proposed by Caporossi et al. Moreover, we determine the graphs with the minimal and second-minimal energies for $n-1\leq e\leq \frac{3n}{2}-3$, and the unique graph with minimal energy for $\frac{3n-5}{2}\leq e\leq 2n-4$ among all quasi-trees with n vertices and e edges, respectively.

Induced subgraphs with large degrees at end–vertices for hamiltonicity of claw–free graphs

Roman Čada[1], Binlong Li[1,2], Bo Ning[2], Shenggui Zhang[2]

[1] Department of Mathematics, NTIS - New Technologies for the Information Society, University of West Bohemia, 30614 Pilsen, Czech Republic

[2] Department of Applied Mathematics, School of Science, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, P.R. China

A graph is called *claw–free* if it contains no induced copy of the claw ($K_{1,3}$). Matthews and Sumner proved that a 2–connected claw-free graph G is hamiltonian if every vertex of it has degree at least $(|V(G)|-2)/3$. On the workshop C&C (Novy Smokovec, 1993), Broersma conjectured the degree condition of this result can be restricted only to end–vertices of induced copies of N (the graph obtained from a triangle by adding three disjoint pendant edges). Fujisawa and Yamashita showed that the degree condition of Matthews and Sumner can be restricted only to end–vertices of induced copies of Z_1 (the graph obtained from a triangle by adding one pendant edge). Our main result in this paper is a characterization of all graphs H such that a 2–connected claw–free graph G is hamiltonian if each end-vertex of every induced copy of H in G has degree at least $|V(G)|/3+1$. This gives an affirmation of the conjecture of Broersma up to an additive constant.

A note on induced subtrees and chromatic number of a graph

Yingli Zhang

Nanjing Normal University

Gyárfás conjectured that for every tree T, there exists an integer-valued function $f_T(x)$ such that every graph G of chromatic number larger than $f_T(\omega(G))$ contains T as an induced subgraph, where $\omega(G)$ is the clique number of G. In this note, we generalize two results of Gyárfás, Szemerédi and Tuza.

Strongly symmetric self–orthogonal diagonal Latin squares and Yang Hui type magic squares

Yong Zhang

School of Mathematical Sciences, Yancheng Teachers University, Jiangsu 224002, China

Let A be a magic square of even order n. A is a *Yang Hui type magic square with t-powered sum* if for each $e = 2, 3, \cdots, t$, the sum of the elements of the first $\frac{n}{2}$ rows of A^{*e} is the same as that of the last $\frac{n}{2}$ rows, and the sum of the elements of the left $\frac{n}{2}$ columns of A^{*e} is the same as that of the right $\frac{n}{2}$ columns, where $A^{*e} = (a_{i,j}^e)$. A Yang Hui type magic square is a weak form of a t–multimagic square. In this report, a kind of strongly symmetric self-orthogonal diagonal Latin square is introduced and some constructions of Yang Hui type magic squares are presented based on strongly symmetric self-orthogonal diagonal Latin squares. Consequently, some families of Hui type magic squares are provided.

The log–balancedness of combinatorial sequences

Fengzhen Zhao

Department of Mathematics, Shanghai University, Shanghai 200444, China

For a given sequence $\{z_n\}_{n\geq 0}$ of positive real numbers, $\{z_n\}_{n\geq 0}$ is said to be *log–concave* (or *log–convex*) if $z_n^2 \geq z_{n-1}z_{n+1}$ (or $z_n^2 \leq z_{n-1}z_{n+1}$) for all $n \geq 1$ and $\{z_n\}_{n\geq 0}$ is said to be *log–balanced* if $\{z_n\}_{n\geq 0}$ is log-convex and $\{z_n/n!\}_{n\geq 0}$ is log-concave. Log–concavity and log–convexity are instrumental in obtaining the growth rate of a sequence and they are also sources of inequalities. It is clear that a sequence $\{z_n\}_{n\geq 0}$ is log–convex (log–concave) if and only if its quotient sequence $\{z_{n+1}/z_n\}_{n\geq 0}$ is nondecreasing (nonincreasing). In fact, a log–balanced sequence is log–convex, but its quotient sequence does not grow too fast.

In this paper, we mainly introduce several sufficient conditions for a combinatorial sequence given by a three–term linear homogeneous recurrence to be log–balanced. In particular, we discuss the log–balancedness of the Catalan–Larcombe–French sequence.

On the (signless) Laplacian eigenvalues of graphs with given stablity number

Qin Zhao

Faculty of Mathematics & Statistics, Hubei University, Wuhan 430062, China

Let G be a simple undirected graph with n vertices. A *stable set* of a graph G is a set of vertices no two of which are adjacent. The cardinality of a maximum stable set in a graph G is called the *stability number* of G and is denoted by $\alpha(G)$. In this talk, I will introduce some results on the (signless) Laplacian eigenvalues of graphs with given stablity number $\alpha \geq \lceil \frac{n}{2} \rceil$. we determine the graph which has the minimum least signless Laplacian eigenvalue among all nonbipartite graphs with given stability number $\alpha \geq \lceil \frac{n}{2} \rceil$ and the graph which has the minimum algebraic connectivity among all connected graphs with given stability number $\alpha \geq \lceil \frac{n}{2} \rceil$, respectively.

On S–(p, q)–Dyck paths

Tongyuan Zhao

School of Mathematics, Peking University

Bizley studied a generalization of Dyck paths from $(0, 0)$ to (pd, qd) $(\gcd(p, q) = 1)$, which never go below the line $py = qx$ with step set $\{(0, 1), (1, 0)\}$, and enumerated such paths. In this talk we generalize Bizley's results to the *S–(p, q)–Dyck paths*, i.e., paths with an arbitrary given step set S. Explicit formulas are obtained by modified Gessel's method, which are also appropriate for higher dimension cases. As applications, we calculate some examples that contain generalizations of the classical Schröder and Motzkin numbers.

Competition numbers of Halin graphs

Yongqiang Zhao

School of Mathematics and Information Science, Shijiazhuang University, Shijiazhuang 050035, P. R. China

For any graph G, G together with sufficiently many isolated vertices is the competition graph of some acyclic digraph. The competition number $k(G)$ of a graph G is defined to be the smallest number of such isolated vertices. In general, it is hard to compute the competition number $k(G)$ for a graph G and characterizing a graph by

its competition number has been one of important research problems in the study of competition graphs. A Halin graph is a planar graph consisting of a tree with no vertex of degree two and a cycle connecting the leaves of the tree.

In this paper, we compute the competition numbers of Halin graphs.

Line star sets for Laplacian eigenvalues

Jiang Zhou

College of Science, Harbin Engineering University, Harbin 150001, PR China

Let G be a graph with a nonzero Laplacian eigenvalue μ of multiplicity k. A line star set for μ in G is a set Y of k edges of G such that μ is not a Laplacian eigenvalue of $G--Y$. It is not difficult to see that line star set exists for any nonzero Laplacian eigenvalue of any graph.

In this talk, we give some results on line star sets for Laplacian eigenvalues.

Proofs of conjectures of Sandon and Zanello on colored partition identities

Rui Zhou

Northeastern University at Qinhuangdao

In a recent systematic study, C. Sandon and F. Zanello offered 30 conjectured identities for partitions. In this lecture, we provide proofs for all 30 conjectures of Sandon and Zanello. Most of our proofs depend upon known modular equations and formulas of Ramanujan for theta functions, while for the remainder of our proofs it was necessary to derive new modular equations and to employ the process of duplication to extend Ramanujan's catalogue of theta function formulas.

Some recent results for unimodality of combinatorial sequences

Baoxuan Zhu

School of Mathematics and Statistics, Jiangsu Normal University, Xuzhou, China

Unimodality problems often arise in combinatorics, analysis, algebra, geometry, probability and statistics. In this topic, we will report some recent advance in unimodality results of combinatorial sequences.