

Self-induced Anderson localization and optical limiting in photonic crystal coupled cavity waveguides with Kerr nonlinearity

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The transmission behavior of photonic crystal coupled cavity waveguides (CCWs) with Kerr nonlinearity is investigated by numerical simulations based on the finite-difference time-domain technique. The authors find that a nearly ideal optical limiter can be realized by use of a nonlinear CCW. In addition, it is revealed that Anderson localization [Phys. Rev. **109**, 1492 (1958)] of the extended states in the impurity band instead of the shift of the impurity band is responsible for the observed optical limiting. Therefore, nonlinear CCWs offer a convenient platform for studying Anderson localization of electromagnetic waves in a controlled fashion and will find potential applications in optical limiting and switching. © 2007 American Institute of Physics.
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Anderson localization of electronic waves in disordered materials has been of great interest in the last 40 years.¹ It initially studied the propagation behavior of electronic waves in disordered solids. With the advance in the fabrication of artificial crystals, the study of Anderson localization has been extended to semiconductor superlattices.² On the other hand, it has been realized that classical waves such as electromagnetic (EM) waves or elastic waves may become localized in a medium whose dielectric function or elastic coefficient, respectively, varies randomly in space.³ In fact, Anderson localization of light in a medium composed of nanocrystals randomly distributed in a matrix or in a disordered two-dimensional (2D) photonic lattice has been demonstrated experimentally.^{4,5}

For semiconductor superlattices, the localization of electronic waves has been investigated either by intentionally introducing a size fluctuation in the constituent wells² or by applying an electric field on a superlattice without any disorder.⁶ Being the counterparts of semiconductors, some attention has been paid to the localization of EM waves in photonic crystals (PCs) with disorders introduced intentionally or unintentionally.^{5,7,8} However, the localization of EM waves in PCs introduced and therefore controlled by an external field has not yet been studied. Apparently, the counterparts of semiconductor superlattices in PCs are coupled cavity waveguides (CCWs).^{9,10} Similar to the electronic waves in semiconductor superlattices, the EM waves in CCWs are primarily confined in the constituent cavities and the overlap between them in the neighboring cavities allows the propagation of the EM waves along the CCWs.^{9,10} If we introduce nonlinearity into the constituent cavities of the CCWs, e.g., Kerr nonlinearity, then the localization of EM waves can be realized by the electric field of an incident wave. In this

letter, we investigate by numerical simulations the localization of EM waves in nonlinear CCWs and its potential application in optical limiting.

The physical model we used is schematically shown in Fig. 1(a). It is a one-dimensional (1D) CCW formed in a semiconductor strip waveguide with Kerr nonlinearity (e.g., GaAs). Usually, three parameters are employed to describe the configuration of a CCW.¹¹ Here, they are the total number of the cavities N , the number of air holes in between the two cavities n , and that on both sides of the waveguide m . Other structure parameters include the lattice constant $a=0.5 \mu\text{m}$, the radii of the air holes $r=0.2a$, the size of the cavities $d=1.5a$, the width of the waveguide $w=1.2a$, and

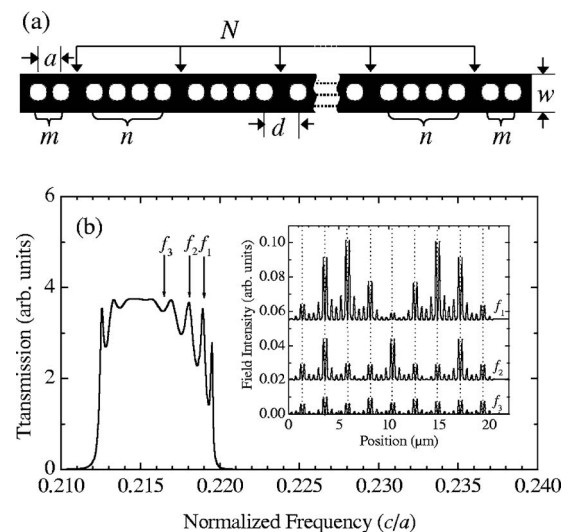


FIG. 1. (a) Structure of the 1D CCW with Kerr nonlinearity investigated in this letter. (b) Linear transmission spectrum of the impurity band for the 1D CCW with $N=9$, $n=4$, and $m=2$. The distributions of the electric field intensity within the CCW at three different frequencies are given in the inset. The dotted lines indicate the positions of the constituent cavities.

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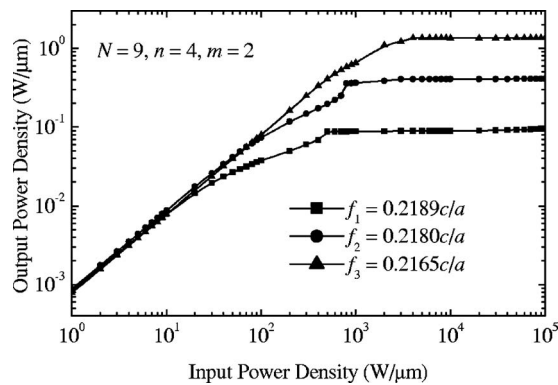


FIG. 2. Dependence of the output power density on the input at three different frequencies indicated in Fig. 1(b) for the nonlinear CCW ($N=9$, $n=4$, and $m=2$).

the linear refractive index of the waveguide $n_0=3.4$. We will use the finite-difference time-domain technique to simulate the transmission behavior of the nonlinear CCW.¹² In addition, we have chosen to do 2D simulations which have been confirmed to be a good approximation of the three-dimensional case.¹³ The grid sizes in both directions are chosen to be $a/10$ and a perfectly matched layer boundary condition is employed. The nonlinear coefficient of the waveguide is chosen to be $n_2=5 \times 10^{-5} \mu\text{m}^2/\text{W}$ (or $5 \times 10^{-13} \text{cm}^2/\text{W}$), which is close to that of GaAs. The linear transmission spectrum of the CCW with $N=9$, $n=4$, and $m=2$ is presented in Fig. 1(b). Also, we show the distributions of the electric field intensity within the CCW at three different frequencies ($f_1=0.2189c/a$, $f_2=0.2180c/a$, and $f_3=0.2165c/a$) in the inset. It is noticed that the field intensities in the constituent cavities are generally different. Besides, such a difference becomes larger when the frequency approaches the band edge. Since the resonant frequency shifts of the cavities depend strongly on the field intensities inside ($\Delta f \propto \Delta n \sim n_2|E|^2$), it implies that there exists an inhomogeneous broadening of the resonant frequencies which is dependent on the input power density.

It has been suggested many years ago that optical limiting would appear in a nonlinear PC due to the shift of the band edges caused by the nonlinearity-induced refractive index change.¹⁴ If the refractive index change over the entire PC is uniform (e.g., the PC is illuminated by a light from the top), then we will be able to observe the shift of the band edges of the PC. As discussed above, however, this is not true for the nonlinear CCW. Since the resonant frequency shifts of the cavities are different, Anderson localization of EM waves is expected to appear in a way similar to that observed in semiconductor superlattices.^{2,6} The inhomogeneous broadening of the cavity modes results in the localization of the extended states in the impurity band, starting from the band edges and eventually spreading over the entire impurity band with increasing inhomogeneous broadening.¹⁵ The localization of the extended states will lead to the reduction in the transmission and in the field intensities inside the cavities that in turn alleviate the localization. Finally, a balance is achieved at every input power density and the corresponding transmitted intensity remains unchanged.

Now let us examine in detail the transmission behavior of the nonlinear CCW. Input waves with frequencies of f_1 , f_2 , and f_3 have been investigated and the dependences of the output power density on the input are presented in Fig. 2.

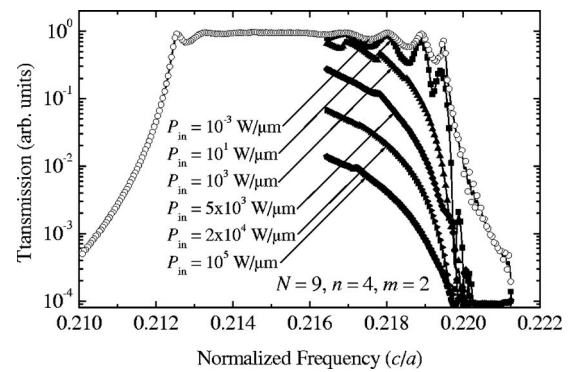


FIG. 3. Evolution of the high-frequency side of the impurity band for the nonlinear CCW ($N=9$, $n=4$, and $m=2$) with increasing input power density.

Interestingly, we observe in all three cases a nearly ideal optical limiting behavior, i.e., the output power density exhibits a linear dependence on the input at low densities and it keeps to be a constant after a threshold.¹⁶ As expected, the extended states close to the band edge (e.g., f_1) are localized first, resulting in a smaller threshold and a lower limiting power density. In addition, the threshold and the limiting output power density can also be adjusted by changing the width of the impurity band. In Fig. 2, the most remarkable feature is the constant output power density at high input power densities. This is not observed in the optical limiters reported so far by utilizing other mechanisms.¹⁷⁻²⁰

In order to confirm that Anderson localization of EM waves is really responsible for the optical limiting, we show in Fig. 3 the evolution of the impurity band with increasing input power density. Only the high-frequency side of the impurity band is presented because a modulation instability appears in the low-frequency side with an anomalous dispersion.²¹ It is apparent that the localization of the extended states instead of the shift of the impurity band is the major mechanism of optical limiting. Moreover, it is noticed that the ripples observed in the impurity band disappear at high densities because of the enhancement of nonlinearity at the peaks of the ripples which results in a fast reduction of the transmission.

It is necessary to address the difference between the localization caused by structure disorders and that induced by nonlinearity studied here. In the former case, disorders of different extents are introduced externally and they are independent of the input wave. Even in Wannier-Stark localization, the disorders are generated by external electric fields applied on superlattices.⁶ The investigation of such localization is generally carried out by evaluating the dependence of transmission on sample size.^{4,7,8} For nonlinear CCWs, however, the disorder is introduced through the internal nonlinearity. Due to the tight-binding character of CCWs, the transfer matrix that characterizes the coupling strength between the neighboring cavities changes from site to site.⁹ More importantly, the disorder should be triggered by the input wave and depends strongly on its intensity. This self-induced feature indicates that the dramatic reduction in the transmission due to Anderson localization acts as a strong negative feedback for the input wave and it is quite important for realizing optical limiter with nearly ideal performance. On the other hand, it implies that the exponential decay of the transmission with increasing sample size, which is expected in the former case, will not be manifested in our case. If we employ

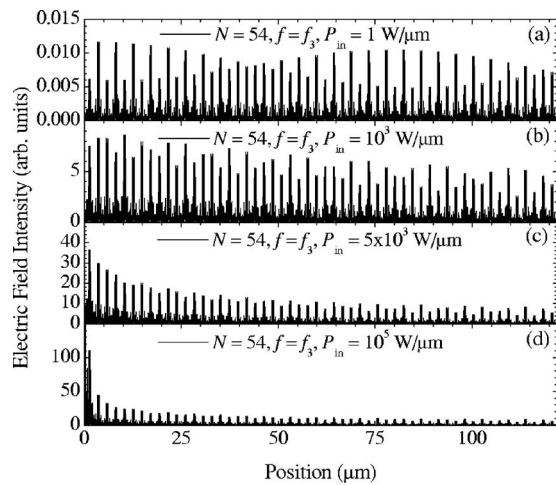


FIG. 4. Distributions of the electric field intensity within the nonlinear CCW with $N=54$, $n=4$, and $m=2$ at four different input power densities. (a) $1 \text{ W}/\mu\text{m}$, (b) $10^3 \text{ W}/\mu\text{m}$, (c) $5 \times 10^3 \text{ W}/\mu\text{m}$, and (d) $10^5 \text{ W}/\mu\text{m}$.

a pump-probe scheme, e.g., introducing disorder at f_3 while detecting the variation of transmission at f_1 , it is possible to observe the sharp decrease of transmission caused by Anderson localization. The scenario like this suggests an all-optical switching mechanism and will be discussed elsewhere.

In principle, the wave function of a 1D disordered system is always localized even for an infinitely small disorder.³ Only when the localization length is reduced to be smaller than the sample size, however, the effect of localization will become significant and effective.^{2,6} As discussed above, we cannot estimate the localization length using the formula given in Refs. 7 and 8 because of the self-induced feature of the disorder in the nonlinear CCW. As an alternative, we can give a rough estimation of the localization length by examining the distribution of the electric field intensity over the nonlinear CCW.¹⁵ In Fig. 4, we show the calculated distributions of the electric field intensity within the CCW with $N=54$, $n=4$, and $m=2$ at four input power densities (P_{in}) of 1 , 10^3 , 5×10^3 , and $10^5 \text{ W}/\mu\text{m}$ for f_3 . They represent four typical conditions with different transmission behaviors, i.e., far below, just below, just above, and far above the threshold. For $P_{\text{in}}=1 \text{ W}/\mu\text{m}$, the nonlinear effect is negligible and the field distribution is similar to the linear case. When P_{in} is raised to $10^3 \text{ W}/\mu\text{m}$, a gradual decrease in the field intensity is clearly observed. As $P_{\text{in}}=5 \times 10^3 \text{ W}/\mu\text{m}$, the field intensity inside the cavities exhibits a biexponential decay and the localization length extracted from the fast decay ($\sim 6 \mu\text{m}$) is much shorter than the waveguide length. When $P_{\text{in}}=10^5 \text{ W}/\mu\text{m}$, we find that the localization length is further reduced to be $\sim 2 \mu\text{m}$, implying a deep localization of EM waves. Totally, we have simulated samples with larger cavity numbers up to $N=108$. The localization and optical

limiting are observed at lower input power densities for longer CCWs.

In summary, we have investigated by numerical simulation the transmission behavior of CCWs with Kerr nonlinearity. It is revealed that Anderson localization of the extended states in the impurity band instead of the shift of the entire impurity band dominates the transmission behavior of nonlinear CCWs. The strong negative feedback provided by Anderson localization results in a nearly ideal optical limiting performance, implying the potential applications of nonlinear CCWs in the construction of good and efficient optical limiters and switches.

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