Application of the multiscale singular perturbation method to nonparaxial beam propagations in free space

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Received March 22, 2007; revised June 26, 2007; accepted June 26, 2007;
posted July 23, 2007 (Doc. ID 81347); published September 24, 2007

Starting from the vector Maxwell's equations and applying the multiscale singular perturbation method, the nonparaxial beam propagation is studied in free space. Two new equations have been derived for transverse and longitudinal electric fields of an arbitrary polarized electromagnetic wave even in the case of tightly focused nonparaxial laser beams. By using the analogy of the optical beam in the space domain and the optical pulse in the time domain, the higher-order diffraction term is introduced. For strongly nonparaxial beams that are characterized by large values of the perturbative parameter, our correction solutions yield an accurate description of the field in the near-field region and are consistent with all other correction results obtained by others in the far-field region. For weakly nonparaxial beams, our correction solutions can be expressed in a very simple form that is proved to be exactly consistent with the solutions obtained by Cao and Deng [J. Opt. Soc. Am. A **15**, 1144 (1998)]. In addition, the lowest-order correction to the paraxial approximation can be found to be in good agreement with the results of Lax *et al.* [Phys. Rev. A **11**, 1365 (1975)] and Seshadri [J. Opt. Soc. Am. A **19**, 2134 (2002)]. © 2007 Optical Society of America

OCIS codes: 260.2110, 350.5500, 140.3430.

1. INTRODUCTION

It is well known that the paraxial wave equation gives an accurate description for wave beams near the axis as long as the beam width w_0 remains larger than the radiation wavelength λ throughout the propagation. However, many light sources, such as some solid-state lasers or semiconductor injection lasers, generate wide-angle beams for which the paraxial approximation is not applicable. The importance of the physical model of focused laser fields has been clearly demonstrated in direct laser-electron scattering [1–3] and must be considered for accurately modeling any high-intensity experiment [4]. Moreover, tightly focused laser beams [5–8], whose width may be less than the radiation wavelength, cannot be described with the paraxial approximation. For these nonparaxial light beams, accordingly, the paraxial approximation theory is no longer valid and some corrections are necessary.

The nonparaxial propagation of electromagnetic wave beams has attracted much attention in optics [1,2,9–28]. In 1975, Lax *et al.* developed a perturbative approach in terms of a small dimensionless parameter $(kw_0)^{-1}$, where k is the wave number [9]. Following this pioneer work, many authors made use of various methods to study the corrections to the paraxial approximation [10–19]. Agrawal and co-workers gave the first-order correction for a Gaussian beam [10] and discussed differences in the various beams under different boundary conditions [13]. A vector theory of the free-space wave propagation was brought forward by Davis [11] for the linearly polarized electromagnetic vector potential. Using the perturbation expansion procedure, Couture and Belanger [12] obtained corrections to the paraxial solution. By applying a scalar approach, corrections to higher-order Gaussian beams were obtained by Takenaka *et al.* [14] and Zauderer [15]. Fourteen years ago, Wünsche [16] introduced two related transition operators that can transform arbitrary solutions of the paraxial wave equation into exact solutions of the Helmholtz wave equation under different boundary conditions. Employing the Fourier transform method, Cao and Deng [17] studied the propagation problem of a general nonparaxial light beam whose zeroth-order field is given. In 2002, Seshadri [18] presented certain explanations and remarks on the entire series of nonparaxial corrections for the fundamental Gaussian beam with reference to the analysis given by Wünsche [16]. All these methods are very effective for building analytical approximants for nonparaxial propagations of optical fields. Unfortunately, they were suitable for dealing with weakly nonparaxial beams whose perturbative parameter is small. When the expanding perturbative parameter becomes greater, i.e., for strongly nonparaxial beams, however, the correction solutions obtained by all the methods mentioned above are divergent, and those methods cease to work. In 2003, Borghi and Santariero [19] showed that the method proposed by Lax could be profitably employed even in the case of extremely nonparaxial beams according to a different resummation scheme introduced by Weniger [29]. The correction terms obtained by Borghi and Santarsiero [19] remain convergent even in the case of extremely nonparaxial beams. Recently, Sepke and Umstadter [5–8] derived the exact analytical solution for

the vector electromagnetic field of Gaussian, flattened Gaussian, and annular Gaussian laser modes by using the angular spectrum method. The solutions obtained by Borghi and Santarsiero [19] and by Sepke and Umstadter [5–8] are suitable for linearly polarized beams. Here, we deal with the propagation problem of a general nonparaxial light beam whose initial field is of arbitrary symmetry by using the multiple-scale method, one of the singular perturbation methods [30], which was used to derive the evolution equations of optical pulses in the femtosecond regime [31,32]. This method can be used to deal with linearly polarized beams, radially polarized beams, etc..

The paper is organized as follows: Section 2 is devoted to a concise summary of base equations. In Secction 3, the multiple-scale method of the singular perturbation methods is applied to dealing with the propagation problem of a general nonparaxial light beam. Accordingly, two new equations are derived for transverse and longitudinal electric fields even in the case of tightly focused nonparaxial laser beams. Some applications of this model are given in Section 4. A discussion is presented in Section 5, and simple numerical examples are shown to place in evidence the effectiveness of our solutions. This paper concludes in Section 6.

2. BASE EQUATIONS

The Maxwell's equations for the monochromatic complex fields $\vec{E}$ and $\vec{B}$ varying as $\exp(-i\omega_0 t)$ are [9]

$$\nabla \times \vec{E} = i\omega_0 \vec{B}, \tag{1a}$$

$$\nabla \times \vec{B} = -i\frac{\omega_0}{c^2}\vec{E}, \tag{1b}$$

$$\nabla \cdot \vec{E} = 0, \tag{1c}$$

$$\nabla \cdot \vec{B} = 0, \tag{1d}$$

where ω_0 is the frequency of a monochromatic pulse, $c = 1/\sqrt{\epsilon_0\mu_0}$ is the speed of light in vacuum, ϵ_0 is the vacuum permittivity, and μ_0 is the vacuum magnetic permeability.

From Eqs. (1a)–(1c), it follows that

$$\nabla \times \nabla \times \vec{E} = (\omega_0/c)^2\vec{E}, \tag{2a}$$

$$\nabla \cdot \vec{E} = 0. \tag{2b}$$

Assuming that the beam propagates along the z axis and introducing $\vec{F}_\perp$ and F_z as the slowly varying parts of the transverse electric field $\vec{E}_\perp$ and the longitudinal electric field E_z, we have $\nabla = \nabla_\perp + \hat{z}\partial_z$ and $\vec{E} = \exp(ikz)(\vec{F}_\perp + \hat{z}F_z)$ [9,19,20]; Eqs. (2a) and (2b) become

$$\nabla_\perp\left(\nabla_\perp \cdot \vec{F}_\perp + \frac{\partial F_z}{\partial z} + ikF_z\right) - \nabla_\perp^2\vec{F}_\perp - \frac{\partial^2\vec{F}_\perp}{\partial z^2} - 2ik\frac{\partial\vec{F}_\perp}{\partial z} = 0, \tag{3a}$$

$$\frac{\partial}{\partial z}(\nabla_\perp \cdot \vec{F}_\perp) + ik\nabla_\perp \cdot \vec{F}_\perp - \nabla_\perp^2 F_z = k^2 F_z, \tag{3b}$$

where $k = \omega_0/c$ is the wave number. Now we introduce the dimensionless coordinates

$$\vec{\rho} = \vec{r}_\perp/w_0, \quad \zeta = z/L, \tag{4}$$

where $\vec{r}_\perp$ is the transverse coordinate vector, $\vec{\rho}$ is the normalized transverse coordinate vector, $L = kw_0^2$ is the diffraction length, and w_0 is the beam width, which can be obtained by using the definition of the second-order moment. By substituting Eq. (4) into Eqs. (3a) and (3b), we obtain

$$\nabla_t\left(\sigma\nabla_t \cdot \vec{F}_t + \sigma^2\frac{\partial F_\zeta}{\partial\zeta} + iF_\zeta\right) - \sigma\nabla_t^2\vec{F}_t - \sigma^3\frac{\partial^2\vec{F}_t}{\partial\zeta^2} - 2i\sigma\frac{\partial\vec{F}_t}{\partial\zeta} = 0, \tag{5a}$$

$$\sigma^3\frac{\partial}{\partial\zeta}(\nabla_t \cdot \vec{F}_t) + i\sigma\nabla_t \cdot \vec{F}_t - \sigma^2\nabla_t^2 F_\zeta = F_\zeta, \tag{5b}$$

where $\nabla_t = w_0\nabla_\perp$, $\sigma = w_0/L = (kw_0)^{-1}$ is a dimensionless perturbation parameter, $\vec{F}_\perp(\vec{r}_\perp, z) \to \vec{F}_t(\vec{\rho}, \zeta)$, and $F_z(\vec{r}_\perp, z) \to F_\zeta(\vec{\rho}, \zeta)$.

3. MULTISCALE SINGULAR PERTURBATION METHODS

In Eqs. (5a) and (5b), the independent variable ζ is transformed into several variables by [30,31]

$$\zeta_n = \sigma^n\zeta, \qquad n = 0,1,2,\ldots, \tag{6}$$

where each ζ_n is an order of σ smaller than the previous one. Thus the derivatives should be replaced by the expansion

$$\partial_\zeta = \partial_{\zeta_0} + \sigma\partial_{\zeta_1} + \sigma^2\partial_{\zeta_2} + \cdots. \tag{7}$$

At the same time, $\vec{F}_t$ and F_ζ can be expanded in an asymptotic series

$$\vec{F}_t = \vec{F}_{t(0)} + \sigma\vec{F}_{t(1)} + \sigma^2\vec{F}_{t(2)} + \cdots, \tag{8a}$$

$$F_\zeta = F_{\zeta(0)} + \sigma F_{\zeta(1)} + \sigma^2 F_{\zeta(2)} + \cdots. \tag{8b}$$

From Eqs. (A17) and (A18) in Appendix A, one can obtain the transverse electric field $\vec{E}_{t(2N)}$ after considering the $(2N)$th-order correction term and the longitudinal electric field $E_{\zeta(2N+1)}$ after considering the $(2N+1)$th-order correction term,

$$\vec{E}_{t(2N)}(\vec{\rho},\zeta) = \frac{1}{4\pi^2}\int \vec{A}_0(\zeta = 0)\exp(i\varphi_1)\mathrm{d}^2\vec{k}_t, \tag{9a}$$

$$E_{\zeta(2N+1)}(\vec{\rho},\zeta)=\frac{-\sigma}{4\pi^2}\int \vec{k}_t\cdot\vec{A}_0(\zeta=0)m_2\exp(i\varphi_1)\mathrm{d}^2\vec{k}_t, \tag{9b}$$

where $\vec{A}_0(\zeta=0)=\vec{A}_0(\zeta_{0,1,2,\ldots,2N}=0)$, $\vec{A}_0(\zeta_{0,1,2,\ldots,2N}=0)=\tilde{\vec{F}}_{t(0)}\times(\vec{k}_t,\zeta=0)$ is the Fourier transform of the initial field, k_t is a dimensionless quantity is given by the transverse component of the wave vector times w_0, $\varphi_1=\vec{k}_t\cdot\vec{\rho}+(1-m_1)\zeta/\sigma^2-\omega_0 t$, $m_1=k_t^2\sigma^2/2+\Sigma_{n=2}^{N+1}(2n-3)!!/(2n)!!k_t^{2n}\sigma^{2n}$, $m_2=1+k_t^2\sigma^2/2+\Sigma_{n=2}^{N}(2n-1)!!/(2n)!!k_t^{2n}\sigma^{2n}$, $0!!=1$, and $1!!=1$. Equations (9a) and (9b) can be used to deal with arbitrary symmetric free-propagation beams, such as linearly polarized beams and radially polarized beams. When the parameter N becomes infinity, Eqs. (9a) and (9b) can be changed into

$$\vec{E}_t(\vec{\rho},\zeta)=\frac{1}{4\pi^2}\int \vec{A}_0(\zeta=0)\exp(i\varphi_2)\mathrm{d}^2\vec{k}_t, \tag{10a}$$

$$E_\zeta(\vec{\rho},\zeta)=\frac{-\sigma}{4\pi^2}\int \frac{\vec{k}_t\cdot\vec{A}_0(\zeta=0)}{\sqrt{1-k_t^2\sigma^2}}\exp(i\varphi_2)\mathrm{d}^2\vec{k}_t, \tag{10b}$$

where $\varphi_2=\vec{k}_t\cdot\vec{\rho}+\sqrt{1-k_t^2\sigma^2}\zeta/\sigma^2-\omega_0 t$. Equations (10a) and (10b) are the exact solutions of Maxwell's equations. Equation (10a) is consistent with Eq. (3) of Ref. [27], which was obtained by using the vector angular spectrum method.

It is worth mentioning that, for the nonparaxial laser, partial derivation of Eq. (A17) in Appendix A with respect to ζ gives the expression $\partial\tilde{\vec{E}}_{t(2N)}(\vec{k}_t,\zeta)/\partial\zeta$; inverse Fourier transforming this expression yields a new equation for the transverse component $\vec{E}_{t(2N)}(\vec{\rho},\zeta)$ of the electric vector $\vec{E}$ for the $(2N)$th-order correction term:

$$\frac{\partial\vec{E}_{t(2N)}(\vec{\rho},\zeta)}{\partial\zeta}=\frac{i}{2}\nabla_t^2\vec{E}_{t(2N)}(\vec{\rho},\zeta)-i\sum_{n=2}^{N+1}(-1)^n\times\frac{(2n-3)!!}{(2n)!!}\sigma^{(2n-2)}(\nabla_t^2)^n\vec{E}_{t(2N)}(\vec{\rho},\zeta). \tag{11}$$

By inverse Fourier transforming Eq. (A18), a new equation for the longitudinal component $E_{\zeta(2N+1)}$ of the electric vector $\vec{E}$ for the $(2N+1)$th-order correction term can be obtained,

$$E_{\zeta(2N+1)}(\vec{\rho},\zeta)=i\sigma\left[1+\frac{\sigma^2}{2}\nabla_t^2+\sum_{m=2}^{N}\frac{(2m-1)!!}{(2m)!!}\times\sigma^{2m}(\nabla_t^2)^m\right]\nabla_t\cdot\vec{E}_{t(2N)}(\vec{\rho},\zeta). \tag{12}$$

Now let us examine one special case of Eqs. (11) and (12), for the case of $N=1$; i.e., for the lowest-order transverse and longitudinal correction cases, Eqs. (11) and (12) can be simplified to

$$\frac{\partial\vec{E}_{t(2)}(\vec{\rho},\zeta)}{\partial\zeta}=\frac{i}{2}\nabla_t^2\vec{E}_{t(2)}(\vec{\rho},\zeta)-\frac{i\sigma^2}{8}\nabla_t^4\vec{E}_{t(2)}(\vec{\rho},\zeta), \tag{13a}$$

$$E_{\zeta(3)}(\vec{\rho},\zeta)=i\sigma\left(1+\frac{\sigma^2}{2}\nabla_t^2\right)\nabla_t\cdot\vec{E}_{t(2)}(\vec{\rho},\zeta). \tag{13b}$$

Equation (13a) is the equation for the transverse component $\vec{E}_{t(2N)}$ of the electric vector $\vec{E}$ after considering the second-order correction term. If the the electric vector $\vec{E}$ is linearly polarized, Eq. (13a) is identical to the linear part of Eq. (13) in [33]. Equation (13b) is the equation for the longitudinal component $E_{\zeta(2N+1)}$ of the electric vector $\vec{E}$ after considering the third-order correction term. In our original unscaled variables, Eqs. (11) and (12) yield

$$\frac{\partial\vec{E}_{\perp(2N)}(\vec{r},z)}{\partial z}=\frac{i}{2k}\nabla_\perp^2\vec{E}_{\perp(2N)}(\vec{r},z)-i\sum_{n=2}^{N+1}(-1)^n\times\frac{(2n-3)!!}{(2n)!!}\frac{(\nabla_\perp^2)^n}{k^{(2n-1)}}\vec{E}_{t(2N)}(\vec{r},z), \tag{14a}$$

$$E_{z(2N+1)}(\vec{r},z)=\frac{i}{k}\left[1+\frac{1}{2k^2}\nabla_\perp^2+\sum_{m=2}^{N}\frac{(2m-1)!!}{(2m)!!}\frac{(\nabla_\perp^2)^m}{k^{2m}}\right]\nabla_\perp\cdot\vec{E}_{\perp(2N)}(\vec{r},z). \tag{14b}$$

Equations (14a) and (14b) suggest that the nonparaxial beam propagations of an arbitrary polarized electromagnetic wave can be expressed in terms of the paraxial wave equation plus an additional series of corrective terms.

4. SOME APPLICATIONS OF THIS MODEL

In this section, we will deal with the nonparaxial propagation of a linearly polarized field in free space by using our solutions, Eqs. (9a) and (9b).

We assume the field is linearly polarized in the x direction so that $\vec{F}_\perp=\hat{x}\phi(\vec{r}_\perp,z)$ and $F_z=\psi(\vec{r}_\perp,z)$; at the $z=0$ plane, the Dirichlet boundary condition $\vec{F}_\perp(\vec{r}_\perp,0)=\hat{x}\phi_0(\vec{r}_\perp)$ is satisfied, where $\phi_0(\vec{r}_\perp)$ is an arbitrary complex function. The asymmetric transverse electric and magnetic fields are defined as the electromagnetic field components that lack symmetry in the electric and magnetic fields, since $\vec{F}_y$ is always equal to zero. From Eqs. (9a) and (9b), the asymmetric transverse electric field $E_{x(2N)}^a$ for the $(2N)$th-order correction term and the asymmetric longitudinal electric field $E_{z(2N+1)}^a$ for the $(2N+1)$th-order correction term can be expressed as

$$E_{x(2N)}^a(\xi,\eta,\zeta)=\frac{1}{4\pi^2}\int\int_{-\infty}^{+\infty}\tilde{\phi}_0\exp(i\varphi_1)\mathrm{d}k_\xi\mathrm{d}k_\eta, \tag{15a}$$

$$E_{z(2N+1)}^a(\xi,\eta,\zeta)=-\frac{\sigma}{4\pi^2}\int\int_{-\infty}^{+\infty}k_\xi\tilde{\phi}_0 m_2\exp(i\varphi_1)\mathrm{d}k_\xi\mathrm{d}k_\eta, \tag{15b}$$

where

$$\tilde{\phi}_0 = \int\int_{-\infty}^{+\infty} \phi_0 \exp[-i(k_\xi \xi + k_\eta \eta)] \mathrm{d}\xi \mathrm{d}\eta. \tag{16}$$

When N becomes infinity, it is then easy to show that the asymmetric transverse electric field E_x^a and the asymmetric longitudinal electric field E_z^a can be obtained from Eqs. (15a) and (15b)

$$E_x^a = \frac{1}{4\pi^2} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{17a}$$

$$E_z^a = -\frac{\sigma}{4\pi^2} \int\int_{-\infty}^{+\infty} \frac{k_\xi \tilde{\phi}_0 \exp(i\varphi_2)}{\sqrt{1-k_t^2\sigma^2}} \mathrm{d}k_\xi \mathrm{d}k_\eta. \tag{17b}$$

Equations (17a) and (17b) are consistent with Eqs. (2) and (3) of [1] and Eqs. (3) and (5) of [2]. Equations (17a) and (17b) can be changed into Eqs. (2) and (3) of [5] if the initial field is chosen as the general flattened Gaussian distribution. The other components of the fields can then be calculated by using the Maxwell equations. The symmetric transverse electric and magnetic fields are defined as the electric and the magnetic field components that are symmetric in the electric and magnetic fields. By imposing a boundary condition identical to $\vec{B}(\vec{r}, z=0)$, repeating the same procedures, and averaging the results [2,5–8,24], the symmetric transverse electric and magnetic fields can be obtained:

$$E_{x(2N)} = \frac{1}{8\pi^2} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 (2 - m_1 + \sigma^2 k_\eta^2 m_2) \exp(i\varphi_1) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{18a}$$

$$E_{y(2N)} = -\frac{\sigma^2}{8\pi^2} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 k_\xi k_\eta m_2 \exp(i\varphi_1) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{18b}$$

$$E_{z(2N+1)} = -\frac{\sigma}{8\pi^2} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 k_\xi (1 + m_2) \exp(i\varphi_1) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{18c}$$

$$B_{x(2N)} = \frac{E_{y(2N)}}{c}, \tag{18d}$$

$$B_{y(2N)} = \frac{1}{8\pi^2 c} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 (2 - m_1 + \sigma^2 k_\xi^2 m_2) \exp(i\varphi_1) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{18e}$$

$$B_{z(2N+1)} = -\frac{\sigma}{8\pi^2 c} \int\int_{-\infty}^{+\infty} \tilde{\phi}_0 k_\eta (1 + m_2) \exp(i\varphi_1) \mathrm{d}k_\xi \mathrm{d}k_\eta. \tag{18f}$$

When the parameter N becomes infinity, the electric and the magnetic field components are then

$$E_x = \frac{1}{8\pi^2} \int\int_{-\infty}^{+\infty} \left(1 + m_3 + \frac{k_\eta^2 \sigma^2}{m_3}\right) \tilde{\phi}_0 \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{19a}$$

$$E_y = -\frac{\sigma^2}{8\pi^2} \int\int_{-\infty}^{+\infty} \frac{k_\xi k_\eta \tilde{\phi}_0}{m_3} \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{19b}$$

$$E_z = -\frac{\sigma}{8\pi^2} \int\int_{-\infty}^{+\infty} k_\xi \left(1 + \frac{1}{m_3}\right) \tilde{\phi}_0 \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{19c}$$

$$B_x = \frac{E_y}{c}, \tag{19d}$$

$$B_y = \frac{1}{8c\pi^2} \int\int_{-\infty}^{+\infty} \left(1 + m_3 + \frac{k_\xi^2 \sigma^2}{m_3}\right) \tilde{\phi}_0 \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{19e}$$

$$B_z = -\frac{\sigma}{8c\pi^2} \int\int_{-\infty}^{+\infty} k_\eta \left(1 + \frac{1}{m_3}\right) \tilde{\phi}_0 \exp(i\varphi_2) \mathrm{d}k_\xi \mathrm{d}k_\eta, \tag{19f}$$

where $m_3 = \sqrt{1-k_t^2\sigma^2}$. Equations (19a)–(19f) are the exact integral solutions of the six electromagnetic field components. If we take the initial field as a Gaussian profile with a beam waist w_0, $E_x(x,y,z=0) = E_0 \exp[-(x^2+y^2)/w_0^2]$, and we can write its transverse Fourier transform as $\tilde{\phi}_0 = \pi E_0 \exp[-(k_\xi^2 + k_\eta^2)/4]$; Eqs. (19a)–(19f) can be changed into

$$E_x = \frac{E_0}{4}\left(I_1 + \sigma^2 \frac{\xi^2 - \eta^2}{r_1^3} I_2 + \sigma^2 \frac{\eta^2}{r_1^2} I_3\right), \tag{20a}$$

$$E_y = -\frac{\sigma^2 E_0}{4} \frac{\xi\eta}{r_1^3} (r_1 I_3 - 2I_2), \tag{20b}$$

$$E_z = -\frac{i\sigma E_0}{4} \frac{\xi}{r_1} I_4, \tag{20c}$$

$$B_x = \frac{E_y}{c}, \tag{20d}$$

$$B_y = \frac{E_0}{4c}\left(I_1 + \sigma^2 \frac{\eta^2 - \xi^2}{r_1^3} I_2 + \sigma^2 \frac{\xi^2}{r_1^2} I_3\right), \tag{20e}$$

$$B_z = -\frac{i\sigma E_0}{4c} \frac{\eta}{r_1} I_4, \tag{20f}$$

where

$$I_1 = \int_0^{1/\sigma} (1 + \sqrt{1-k_t^2\sigma^2}) \exp\left(-\frac{k_t^2}{4} - i\varphi_5\right) J_0(k_t\rho) k_t \mathrm{d}k_t, \tag{21a}$$

$$I_2 = \int_0^{1/\sigma} \frac{1}{\sqrt{1-k_t^2\sigma^2}} \exp\left(-\frac{k_t^2}{4} - i\varphi_5\right) J_1(k_t\rho) k_t^2 \mathrm{d}k_t, \tag{21b}$$

$$I_3 = \int_0^{1/\sigma} \frac{1}{\sqrt{1-k_t^2\sigma^2}} \exp\left(-\frac{k_t^2}{4} - i\varphi_5\right) J_0(k_t\rho) k_t^3 \mathrm{d}k_t, \quad (21c)$$

$$I_4 = \int_0^{1/\sigma} \left(1 + \frac{1}{\sqrt{1-k_t^2\sigma^2}}\right) \exp\left(-\frac{k_t^2}{4} - i\varphi_5\right) J_1(k_t\rho) k_t^2 \mathrm{d}k_t, \quad (21d)$$

$\varphi_5 = \omega_0 t - \sqrt{1-k_t^2\sigma^2}\zeta/\sigma^2$, and $J_n(x)$ is the nth-order Bessel function of the first kind. The real part of Eqs. (20a)–(20f) is consistent with Eqs. (7a)–(7f) of [2], which are obtained by using the angular spectrum representation of plane waves. If the initial field is chosen as a general flattened Gaussian distribution with a beam waist w_0, i.e., Eq. (1) of [5], the real part of Eqs. (20a)–(20c) can be transformed into Eqs. (7)—(9) of [5]. A similar approach, using Eqs. (9a) and (9b) as a guide, could presumably be used to deal with the nonparaxial propagation of the radially polarized light beams, the circularly polarized light beams, etc., as desired.

5. DISCUSSION

Now, let us provide a discussion in this section. In Eqs. (9) and (15), if we employ the power expansion $\exp(u) = \Sigma_{m=0}^{\infty} u^m/m!, u = -i\Sigma_{n=2}^{N+1}(2n-3)!!/(2n)!!(k_\xi^2+k_\eta^2)^n \zeta_{2n-2}$ and the inverse Fourier transform relation $\mathcal{F}^{-1}[f_x^2+f_y^2] = -1/(4\pi^2)(\partial^2/\partial x^2+\partial^2/\partial y^2)$, the general correction field $E_{x(2N)}^a$ and $E_{z(2N+1)}^a$ of Eqs. (9a) and (9b) can be expressed as Eqs. (15) and (17) of [28], and the general correction field $E_{x(2N)}^a$ and $E_{z(2N+1)}^a$ of Eqs. (15a) and (15b) can be written in a very simple form that is exactly consistent with the result obtained by Cao and Deng [17]. In this case, Eqs. (18a)–(18f) can be reduced to Eqs. (16a)–(16f) of [2] if the initial field is taken as a Gaussian profile. In addition, the lowest-order correction to the paraxial approximation is found to be in good agreement with the result of Lax *et al.* [9] and Seshadri [18]. Obviously, $\exp(u)$ can be expressed as the sum of the finite terms only if u is a small quantity. However, u is no longer a small quantity as the perturbation parameter σ increases. As a result, the convergence properties of the Lax's series expansions will become worse and worse when the nonparaxial character of the beam is increased. In this case, the Lax's method is not effective. Applying a different resummation scheme recently introduced by Weniger [29] and employing Lax's method [9] preserves the good convergence properties of the correction terms even in case of extremely nonparaxial beams [19]. Recently, Sepke and Umstadter [5] derived the exact analytical solution for the vector electromagnetic field of Gaussian, flattened Gaussian, and annular Gaussian laser modes by using the angular spectrum method. The solutions obtained by Borghi and Santarsiero [19] and by Sepke and Umstadter [5] are suitable for the linearly polarized beams. Here, our solution can be used to deal with linearly polarized beams, radially polarized beams, etc.

It is well known that the group-velocity-dispersion-induced temporal effects [34] for the pulse and the diffraction-induced spatial effects [35] for the beam are a closely analogous pair. Moreover, both of them can be dealt with in Fourier analysis: the temporal frequency of the pulse is the analog of the spatial spectrum of the beam. Using the analogy of the optical beam in the space domain and the optical pulse in the time domain [34,36–38] and comparing Eqs. (14a) and (3.3.1) of [34], we introduce all terms in the last summation term of Eqs. (14a) as the higher-order diffraction terms. The higher the correction term's order N is, the higher the diffraction term's order is. The higher-order diffraction has little effect on the beam propagation in the far-field region because the beam expands in free space, however, the higher-order diffraction has a great effect on the beam propagation in the near-field region, as shown in Figs. 1–4. Equation (14a) shows that the odd-order diffraction terms disappear in the isotropic homogeneous medium. The higher-order diffraction terms can be used to analyze the linear broadening of the beams.

Making a comparison among Lax's lowest-order solution, Borghi's lowest-order solution, our lowest-order solution obtained from Eqs. (18a)–(18f), and the exact solution that can be obtained from Eqs. (20a)–(20f), we use the same input beam parameters, i.e., $\phi(\vec{r},0) = \exp(-r^2/w_0^2)$, $r^2 = x^2+y^2$ for numerical simulations. The normalized amplitudes of the transverse and the longitudinal electric

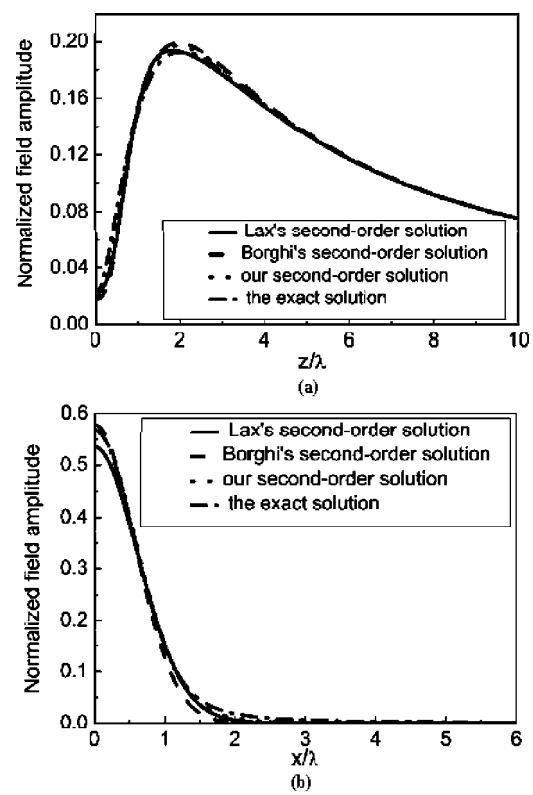

Fig. 1. Behavior of the normalized amplitude of the transverse electric field components as a function of (a) z/λ and (b) x/λ for a Gaussian beam with a spot size $w_0/\lambda = 1/2$ for transverse coordinates $y=0$. Solid curves, Lax's second-order solution; dashed curves, Borghi's second-order solution; dotted curves, our second-order solution; dash-dotted curves, the exact solution. The parameters are chosen as (a) $x=\lambda$, (b) $z=\lambda$. Included are the solutions of four models corresponding to Lax's second-order solution, Borghi's second-order solution, our second-order solution and the exact solution, which are a little bit different and cannot be distinguished by eye when $0.9 < z/\lambda < 1.1$, $z/\lambda > 4$ in (a) and when $0.5 < x/\lambda < 1$ in (b).

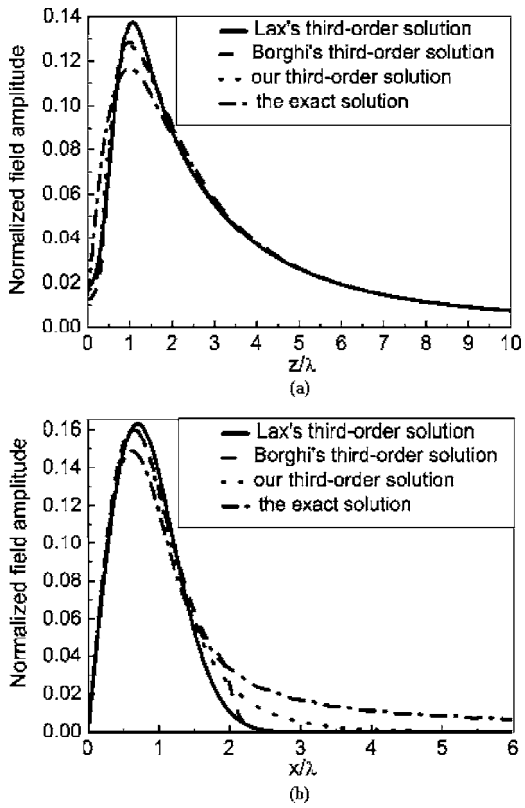

Fig. 2. Behavior of the normalized amplitude of the longitudinal electric field components as a function of (a) z/λ and (b) x/λ for a Gaussian beam with a spot size $w_0/\lambda=1/2$ for transverse coordinates $y=0$. Sold curves, Lax's third-order solution; dashed curves, Borghi's third-order solution; dotted curves, our third-order solution; dash-dotted curves, the exact solution. The parameters are chosen to be the same as those in Fig. 1. Included are the solutions of four models corresponding to Lax's second-order solution, Borghi's second-order solution, our second-order solution, and the exact solution, which are a little bit different and cannot be distinguished by eye when $z/\lambda>3$ in (a) and when $0.0<x/\lambda<0.8$ in (b).

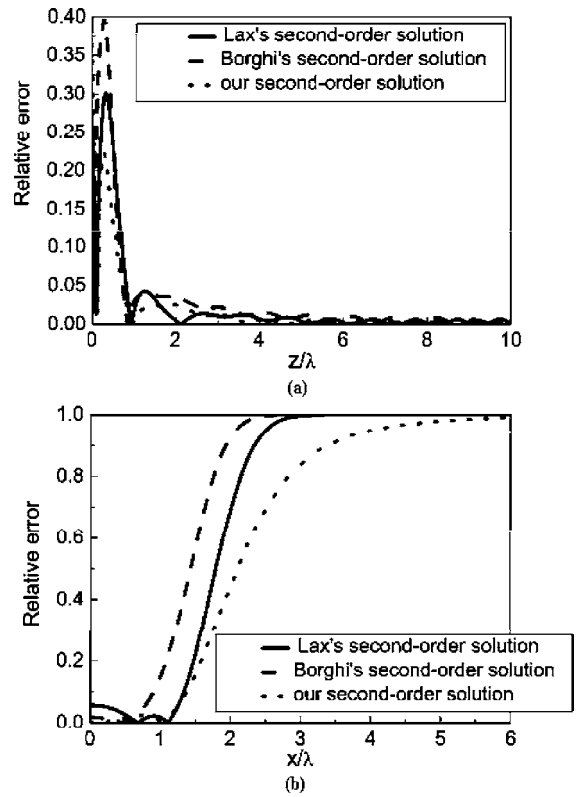

Fig. 3. Behavior of the relative error for the transverse electric field components as a function of (a) z/λ and (b) x/λ for a Gaussian beam with a spot size $w_0/\lambda=1/2$ for transverse coordinates $y=0$. Solid curves, Lax's second-order solution; dashed curves, Borghi's second-order solution; dotted curves, our second-order solution. The parameters are chosen to be the same as those in Fig. 1.

field components are shown as functions of (a) z/λ; in Figs. 1(a) and 2(a) and as functions of x/λ in Figs. 1(b) and 2(b), obtained for the four models for a Gaussian beam with a spot size $w_0/\lambda=1/2$ and the transverse coordinate $y=0$. The behavior of the relative error for the modulus of the transverse and the longitudinal electric fields components, i.e., the quantity $\epsilon(r,z)=|\vec{E}(r,z)^{\text{exact}}-\vec{E}(r,z)^{\text{correction}}|/|\vec{E}(r,z)^{\text{exact}}|$ is plotted in Figs. 3(a) and 4(a) as a function of z/λ and as a function of x/λ in Figs. 3(b) and 4(b) for a Gaussian beam with a spot size $w_0/\lambda=1/2$ and the transverse coordinate $y=0$. Since the beam expands in the transverse plane with the increase of z, the quantitative differences among the four solutions vanish in the far-field region, as presented in Figs. 1(a) and 2(a). It follows from Fig. 3(a) that the maximum value of the relative error of our lowest-order solution is not more than 5.5% if $z>0.8\lambda$ for the transverse electric field. Figure 3(b) shows that the maximum value of the relative error of our lowest-order solution is not more than 4.0% if $x<1.2\lambda$ for the transverse electric field. It can be seen from Fig. 4(a) that the maximum value of the relative error of our lowest-order solution is not more than 8.5% if $z>0.54\lambda$ for the longitudinal electric field. Figure 4(b) shows that the maximum value of the relative error of our lowest-order solution is less than 8.6% if $x<1.65\lambda$ for the longitudinal electric field. The results shown in Figs. 1–4 indicate that in these cases our lowest-order solution is more accurate than those obtained by Lax's model and Borghi's model in dealing with linearly polarized beams.

6. CONCLUSION

In summary, starting from the vector Maxwell's equations, we have studied nonparaxial beam propagation in free space by applying the singular perturbation methods. Two new equations that yield higher-order corrections have been obtained for transverse and longitudinal electric fields even in the case of tightly focused nonparaxial laser beams. In the near-field region, our correction solutions yield an accurate description of the field for strongly nonparaxial beams that are characterized by large values of the perturbative parameter. In the far-field region, our correction solutions are consistent with all other correction results obtained by others because the beam expands in the transverse plane with the increase of z. For weakly nonparaxial beams, our correction solutions can be written in a very simple form that is proved to be exactly consistent with the solutions obtained by Cao and Deng [17]. As well, the lowest-order correction to the paraxial approximation is found to be consistent with the result of Lax *et al.* [9] and Seshadri [18]. In this case, the six elec-

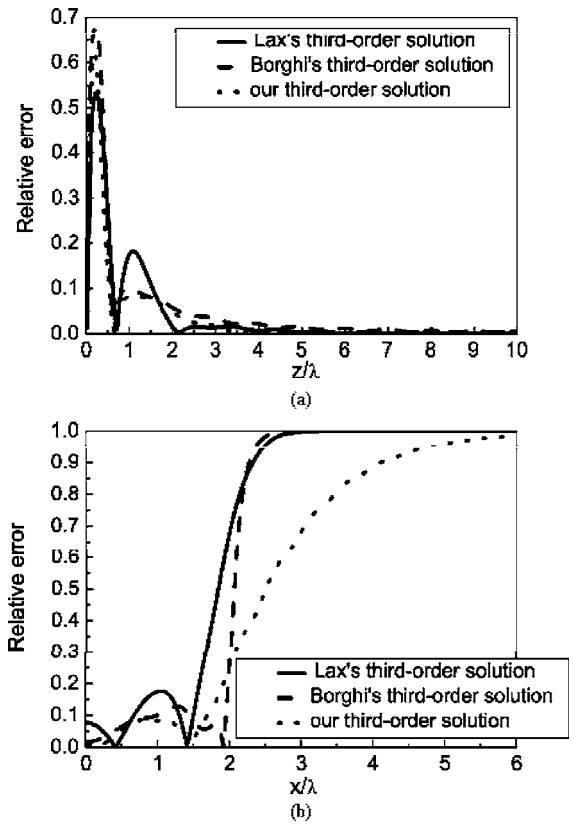

Fig. 4. Behavior of the relative error for the longitudinal electric field components as a function of (a) z/λ and (b) x/λ for a Gaussian beam with a spot size $w_0/\lambda=1/2$ for transverse coordinates $y=0$. Solid curves, Lax's third-order solution; dashed curves, Borghi's third-order solution; dotted curves, our third-order solution. The parameters are chosen to be the same as those in Fig. 1.

tromagnetic field components can be reduced to those results obtained by Quesnel and Mora [2] if the initial field is taken as a Gaussian profile. In addition, this treatment allows us to deal with the nonparaxial propagation of circularly polarized light beams, radially polarized light beams, etc., in the presence of arbitrary symmetric boundary conditions.

APPENDIX A

Substituting Eqs. (7) and (8) into Eqs. (5a) and (5b) and equating the coefficients of each power of σ, one obtains the following equations of different orders:

$$F_{\zeta(0)}=0, \tag{A1}$$

$$\nabla_t^2\vec{F}_{t(0)}+2i\frac{\partial\vec{F}_{t(0)}}{\partial\zeta_0}=0, \tag{A2a}$$

$$F_{\zeta(1)}=i\nabla_t\cdot\vec{F}_{t(0)}, \tag{A2b}$$

$$\nabla_t^2\vec{F}_{t(1)}+2i\left(\frac{\partial\vec{F}_{t(1)}}{\partial\zeta_0}+\frac{\partial\vec{F}_{t(0)}}{\partial\zeta_1}\right)=0, \tag{A3a}$$

$$F_{\zeta(2)}=i\nabla_t\cdot\vec{F}_{t(1)}, \tag{A3b}$$

$$\nabla_t^2\vec{F}_{t(2)}+2i\left(\frac{\partial\vec{F}_{t(2)}}{\partial\zeta_0}+\frac{\partial\vec{F}_{t(1)}}{\partial\zeta_1}+\frac{\partial\vec{F}_{t(0)}}{\partial\zeta_2}\right)=-\frac{\partial^2\vec{F}_{t(0)}}{\partial\zeta_0^2}, \tag{A4a}$$

$$F_{\zeta(3)}=i\nabla_t\cdot\vec{F}_{t(2)}+i\frac{\partial F_{\zeta(1)}}{\partial\zeta_0}, \tag{A4b}$$

$$\nabla_t^2\vec{F}_{t(N)}+2i\sum_{j=0}^{N}\frac{\partial F_{t(N-j)}}{\partial\zeta_j}=-\sum_{j=0}^{[(N-2)/2]}\frac{\partial^2\vec{F}_{t(N-2-2j)}}{\partial\zeta_j^2}-2\sum_{m=0}^{[(N-3)/2]}\sum_{j=0}^{N-(2m+3)}\left(\frac{\partial^2\vec{F}_{t(j)}}{\partial\zeta_m\partial\zeta_{N-2-m-j}}\right), \tag{A5a}$$

$$F_{\zeta(N+1)}=i\nabla_t\cdot\vec{F}_{t(N)}+i\sum_{j=1}^{N-1}\frac{\partial F_{\zeta(j)}}{\partial\zeta_{N-1-j}}\qquad(N>2), \tag{A5b}$$

where square brackets indicate taking the integer. The problem of solving Eqs. (5a) and (5b) for a disturbance $\vec{F}(\vec{\rho},\zeta)$ becomes solving the set of differential equations (A2)–(A5) with the initial conditions

$$\vec{F}_{t(0)}(\vec{\rho},0)=\vec{F}_{t(0)}(\vec{\rho}),\quad \vec{F}_{t(N)}(\vec{\rho},0)=0,\qquad N\geqslant 1, \tag{A6a}$$

$$F_{\zeta(0)}(\vec{\rho},0)=0,\quad F_{\zeta(N)}(\vec{\rho},0)=0,\qquad N\geqslant 1. \tag{A6b}$$

Fourier transforming Eqs. (A2a) and (A2b), it is easy to obtain

$$\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta)=\vec{A}_0(\zeta_0=0,\zeta_1,\ldots)\exp\left(-\frac{ik_t^2\zeta_0}{2}\right), \tag{A7a}$$

$$\tilde{F}_{\zeta(1)}(\vec{k}_t,\zeta)=-\vec{k}_t\cdot\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta), \tag{A7b}$$

where

$$\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta)=\int\vec{F}_{t(0)}(\vec{\rho},\zeta)\exp(-i\vec{k}_t\cdot\vec{\rho})\mathrm{d}^2\vec{\rho}, \tag{A8a}$$

$$\vec{F}_{t(0)}(\vec{\rho},\zeta)=\frac{1}{4\pi^2}\int\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta)\exp(i\vec{k}_t\cdot\vec{\rho})\mathrm{d}^2\vec{k}_t; \tag{A8b}$$

k_t is a dimensionless quantity, and it is given by the transverse component of the wave vector times w_0. $\vec{A}_0(\zeta_0=0,\zeta_1,\ldots)$ is determined by the higher-order perturbed equations, and

$$\tilde{F}_{\zeta(1)}(\vec{k}_t,\zeta)=\int F_{\zeta(1)}(\vec{\rho},\zeta)\exp(-i\vec{k}_t\cdot\vec{\rho})\mathrm{d}^2\vec{\rho}, \tag{A9a}$$

$$F_{\zeta(1)}(\vec{\rho},\zeta)=\frac{1}{4\pi^2}\int\tilde{F}_{\zeta(1)}(\vec{k}_t,\zeta)\exp(i\vec{k}_t\cdot\vec{\rho})\mathrm{d}^2\vec{k}_t. \tag{A9b}$$

From Eqs. (A3a) and (A3b), one can obtain

$$\tilde{\vec{F}}_{t(1)}(\vec{k}_t,\zeta) = -\zeta_0 \frac{\partial \vec{A}_0}{\partial \zeta_1} \exp(-ik_t^2\zeta_0/2), \tag{A10a}$$

$$\tilde{F}_{\zeta(2)}(\vec{k}_t,\zeta) = -\vec{k}_t \cdot \tilde{\vec{F}}_{t(1)}(\vec{k}_t,\zeta). \tag{A10b}$$

When comparing Eq. (A7a) with Eq. (A10a), one can find that $\sigma\tilde{\vec{F}}_{t(1)}(\vec{k}_t,\zeta)$ is a small perturbation of $\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta)$ only when $\sigma\zeta_0=\sigma\zeta$ is a small perturbation, and $\zeta_0\partial\vec{A}_0/\partial\zeta_1 \times\exp(-ik_t^2\zeta_0/2)$ must be zero in order to obtain a higher-order effective expansion expression. For example,

$$\frac{\partial \vec{A}_0}{\partial \zeta_1} = 0. \tag{A11}$$

Equation (A11) shows that $\vec{A}_0$ is independent of ζ_1 explicitly. From Eqs. (A10a) and (A10b), it is easy to derive that $\tilde{\vec{F}}_{t(1)}(\vec{k}_t,\zeta)=0$, $\tilde{F}_{\zeta(2)}(\vec{k}_t,\zeta)=0$. Then, repeating the same procedure, the solutions of Eqs. (A4a) and (A4b) are

$$\tilde{\vec{F}}_{t(2)}(\vec{k}_t,\zeta) = -\zeta_0\left(\frac{\partial \vec{A}_0}{\partial \zeta_2} + \frac{ik_t^4}{8}\vec{A}_0\right)\exp\left(-\frac{ik_t^2\zeta_0}{2}\right), \tag{A12a}$$

$$\tilde{F}_{\zeta(3)}(\vec{k}_t,\zeta) = -\vec{k}_t \cdot \tilde{\vec{F}}_{t(2)}(\vec{k}_t,\zeta) + i\frac{\partial \tilde{F}_{\zeta(1)}(\vec{k}_t,\zeta)}{\partial \zeta_0}. \tag{A12b}$$

Draw a comparison between Eq. (A7a) and Eq. (A12a): it is easy to find that $\sigma^2\tilde{\vec{F}}_{t(2)}(\vec{k}_t,\zeta)$ is a small perturbation of $\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta)$ only when $\sigma^2\zeta_0=\sigma^2\zeta$ is a small perturbation, and $\zeta_0(\partial\vec{A}_0/\partial\zeta_2+ik_t^4\vec{A}_0/8)\exp(-ik_t^2\zeta_0/2)$ must be zero in order to obtain a higher-order effective expansion expression. Say,

$$\frac{\partial \vec{A}_0}{\partial \zeta_2} + \frac{ik_t^4}{8}\vec{A}_0 = 0. \tag{A13}$$

From Eq. (A13), it follows that

$$\vec{A}_0(\zeta_0,\zeta_1,\cdots) = \vec{A}_0(\zeta_{0,1,2}=0,\zeta_3,\cdots)\exp\left(-\frac{ik_t^4\zeta_2}{8}\right). \tag{A14}$$

From Eqs. (A5) and (A7)–(A14), one obtains

$$\tilde{\vec{F}}_{t(3)}(\vec{k}_t,\zeta) = 0, \tag{A15a}$$

$$\tilde{F}_{\zeta(4)}(\vec{k}_t,\zeta) = 0, \tag{A15b}$$

$$\cdots.$$

After a straightforward calculation for the $(2N)$th-order correction terms, one obtains

$$\vec{A}_0(\zeta_0,\zeta_1,\ldots,\zeta_{2N}) = \vec{A}_0(\zeta_{0,1,\ldots,2N}=0) \times \exp\left[-i\sum_{n=2}^{N+1}\frac{(2n-3)!!}{(2n)!!}k_t^{2n}\zeta_{2n-2}\right], \tag{A16}$$

where $\vec{A}_0(\zeta_{0,1,\cdots,2N}=0)=\tilde{\vec{F}}_{t(0)}(\vec{k}_t,\zeta=0)$ is the Fourier transform of the initial field. From Eqs. (A7a), (A10a), (A12a), (A15a), and (A16), the Fourier transforming solution $\tilde{\vec{E}}_{t(2N)}(\vec{k}_t,\zeta)$ of any transverse component of the electric vector $\vec{E}$ after considering the $(2N)$th-order correction term can be written in the form

$$\tilde{\vec{E}}_{t(2N)}(\vec{k}_t,\zeta) = \vec{A}_0(\zeta_{0,1,\ldots,2N}=0)\exp\left(-\frac{im_1\zeta}{\sigma^2}\right), \tag{A17}$$

where $m_1=k_t^2\sigma^2/2+\Sigma_{n=2}^{N+1}(2n-3)!!/(2n)!!k_t^{2n}\sigma^{2n}$, $0!!=1$ and $1!!=1$; from Eqs. (A7b), (A10b), (A12b), (A15b), and (A17), the Fourier transforming solution $\tilde{E}_{\zeta(2N+1)}(\vec{k}_t,\zeta)$ of any longitudinal component of the electric vector $\vec{E}$ after considering the $(2N+1)$th-order correction term can be given by

$$\tilde{E}_{\zeta(2N+1)}(\vec{k}_t,\zeta) = -m_2\sigma\vec{k}_t \cdot \tilde{\vec{E}}_{t(2N)}(\vec{k}_t,\zeta), \tag{A18}$$

where $m_2=1+k_t^2\sigma^2/2+\Sigma_{n=2}^{N}(2n-1)!!/(2n)!!k_t^{2n}\sigma^{2n}$.

ACKNOWLEDGMENTS

This research was supported by the National Natural Science Foundation of China (grants 10474023 and 10674050) and the Specialized Research Fund for the Doctoral Program of Higher Education (grant 20060574006).

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