# Nonlocality-controlled interaction of spatial solitons in nematic liquid crystals

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The authors demonstrate experimentally that interaction between nonlocal solitons in nematic liquid crystals (NLCs) can be controlled by the degree of nonlocality. For a given beam width, the degree of nonlocality can be modulated by changing the pretilt angle $\theta_0$ of NLC molecules through bias voltage $V$. As $V$ increases (so does $\theta_0$), the degree of nonlocality decreases. When the degree of nonlocality is below a critical value, the solitons behave in the way like their local counterpart, i.e., in-phase solitons attract while out-of-phase solitons repulse each other. Such a voltage-controlled interaction between the solitons can be readily implemented in experiments. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2337268]

The interaction properties of two spatial optical solitons depend on the phase difference between them, their coherence,[1,2] and the nonlinear nonlocality of the materials in which the solitons propagate. For a local Kerr-type nonlinearity, two coherent bright solitons attract (or repulse) each other when they are in phase (or out of phase). On the other hand, if the solitons are mutually incoherent[3] or the nonlinear nonlocality of the materials is strong enough,[4–6] the soliton interaction is always attractive, independent of the phase difference. Very recently, Ku *et al.*[2] demonstrated both theoretically and experimentally that the interaction of the solitons can be controlled by varying their total coherence. Thus, the two out-of-phase solitons can repulse or attract each other, depending on whether the coherence parameter is below or above a threshold. Similar dependence of the interaction behavior on nonlocality was also theoretically predicted by Rasmussen *et al.*[7] based on the (1+1)-dimensional model of the nematic liquid crystal (NLC), i.e., there exists a critical degree of nonlocality above which the two out-of-phase solitons will attract each other.

The NLC with a pretilt angle induced by an external low-frequency electric field has been confirmed[8,9] to be a typical material with the strongly nonlocal (referred also as highly nonlocal in some papers[4,8,9]) nonlinearity. In the previous works concerning the single soliton propagation[8–10] and the soliton interaction[5,7,11] in the NLC, however, the peak of the pretilt angle was always made to be $\pi/4$ in order to maximize the nonlinearity. As will be seen later, the degree of nonlocality can only be modulated by changing the beam width in this case, which is not convenient in practice. Recently, Peccianti *et al.* have shown that[12] the nonlocality can be varied by changing the pretilt angle via a bias voltage. In this letter, we use the (1+2)-dimensional model with an arbitrary pretilt angle[12] to describe the (1+2)-dimensional soliton interaction in the NLC. By defining a general characteristic length of the nonlinear nonlocality for the NLC, a voltage-controlled degree of nonlocality is shown to be achieved conveniently. In experiments, we observe the nonlocality-controllable (through the change of the bias voltage) transition from attraction to repulsion of the two out-of-phase solitons in the NLC.

Let us consider the (1+2)-dimensional model of light propagation in a cell containing the NLC. The configuration of the cell and the coordinate system are the same as in the previous works.[8–13] The optical field polarized in $x$ axis with envelope $A$ propagates in $z$ direction. An external low-frequency electric field $E_{\rm rf}$ is applied in $x$ direction to control the initial tilt angle of the NLC. The evolution of the paraxial beam $A$ and optically induced angle perturbation $\Psi$ can be described by the system,[12,13]

$$2ik\partial_z A + \nabla_\perp^2 A + k_0^2\epsilon_a^{\rm op}\sin(2\theta_0)\Psi A = 0, \tag{1}$$

$$\nabla_\perp^2\Psi - \frac{1}{w_m^2}\Psi + \frac{\epsilon_0\epsilon_a^{\rm op}}{4K}\sin(2\theta_0)|A|^2 = 0, \tag{2}$$

where $\theta_0$ and $K$ are the same with that in the Ref. 12, $\nabla_\perp^2=\partial_x^2+\partial_y^2$, $k^2=k_0^2(n_\perp^2+\epsilon_a^{\rm op}\sin^2\theta_0)$, $\epsilon_a^{\rm op}=n_\parallel^2-n_\perp^2$, $\epsilon_a^{\rm rf}=\epsilon_\parallel-\epsilon_\perp$, and the parameter $w_m>0$ for $|\theta_0|\leqslant\pi/2$, which reads

$$w_m = \frac{1}{E_{\rm rf}}\left\{\frac{2\theta_0 K}{\epsilon_0\epsilon_a^{\rm rf}\sin(2\theta_0)[1-2\theta_0\cot(2\theta_0)]}\right\}^{1/2}. \tag{3}$$

In Eq. (2), the term $\partial_z^2\Psi$ has been canceled out because the dependence of $\Psi$ on $z$ is proven to be negligible.[7,13]

Introducing the normalization that $X=x/w_0$, $Y=y/w_0$, $Z=z/(2kw_0^2)$, $a=A/A_0$, and $\psi=\Psi/\Psi_0$, where $A_0=4\sqrt{\pi K/\epsilon_0}/k_0\epsilon_a^{\rm op}w_0^2$, $\Psi_0=\sin(2\theta_0)/k_0^2w_0^2\epsilon_a^{\rm op}$, and $w_0$ the initial beam width, we have the dimensionless system,

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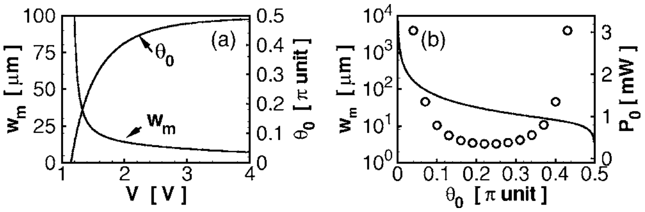

FIG. 1. (a) The characteristic length $w_m$ and the pretilt angle $\theta_0$ of the NLC vs the bias voltage $V$. (b) The characteristic length $w_m$ (a solid curve) and the critical power of a single soliton (circles) vs the pretilt angle $\theta_0$. The parameters are for a 80-$\mu$m-thick cell filled with the NLC (TEB30A) and the critical power is a numerical result.

$$i\partial_Z a + \nabla^2_{XY} a + \gamma\psi a = 0, \tag{4}$$

$$\nabla^2_{XY}\psi - \alpha^2\psi + 4\pi|a|^2 = 0, \tag{5}$$

where $\nabla^2_{XY}=\partial^2_X+\partial^2_Y$, $\gamma=\sin^2(2\theta_0)$, and $\alpha=w_0/w_m$. For a symmetrical geometry, Eq. (5) has a particular solution $\psi(x,y)=(4\pi/\alpha^2)\int\int R(x-x',y-y')|a(x',y')|^2dx'dy'$, and $R(x,y)=(\alpha^2/2\pi)K_0(\alpha\sqrt{x^2+y^2})$, where $K_0$ is the zeroth order modified Bessel function.

We define $w_m$ in Eq. (3) as the general characteristic length of the nonlinear nonlocality for the NLC,[14] then it is obvious that the factor $\alpha$ in Eq. (5) indicates the degree of nonlocality, as defined in Ref. 15 for the specific case of Gaussian response function. A monotonous function of $\theta_0$ on $E_{\rm rf}$ can be approximated as[12] $\theta_0\approx(\pi/2)[1-(E_{\rm Fr}/E_{\rm rf})^3]$ when $E_{\rm rf}$ is higher than the Freèderichsz threshold $E_{\rm Fr}$. Therefore, we can clearly observe from Eq. (3) that $w_m$ is determined only by $E_{\rm rf}$ (or by the bias $V$), or equivalently by the peak-pretilt angle $\theta_0$ for a given NLC cell configuration, as shown in Fig. 1. When the bias is properly chosen so that $\theta_0=\pi/4$,[8] $w_m$ is fixed and $\alpha$ can be modulated only by changing $w_0$. This is the case discussed in Ref. 7. With the decrease of the bias, $\theta_0$ goes from $\pi/2$ to 0, then $\alpha$ varies from $\infty$ to 0 for a fixed $w_0$ and the degree of nonlocality increases from locality to strong nonlocality. As a result, the voltage-controlled degree of nonlocality through the medium of the pretilt $\theta_0$ of the NLC can be achieved conveniently. Figure 1(b) also shows that the critical power reaches its minimum nearby $\theta_0=\pi/4$, and as the pretilt angle approaches $\pi/2$, the critical power increases sharply.

To show the influence of the pretilt angle $\theta_0$ (or equivalently the degree of nonlocality for a fixed $w_0$) on the interaction between the two solitons, we have carried out numerical simulations directly based on the original equations [Eqs. (1) and (2) in Ref. 13]. The simulation results show that there exist critical values for the degree of nonlocality below (or above) which two out-of-phase solitons will repulse (or attract) each other. The critical values depend on the initial separation and relative angle in the $(y,z)$ plane between the solitons. These results agree with the prediction based on the (1+1)-dimensional model.[7] The critical degree of nonlocality for the two parallel solitons is very weak so that the corresponding critical pretilt angle $\theta_{0c}$ is very close to $\pi/2$, leading to a very high critical power for the soliton state [see Fig. 1(b)]. However, the use of a relative angle will significantly increase the critical degree of nonlocality and make $\theta_{0c}$ not be close to $\pi/2$. Hence, the critical powers for different pretilt angles around $\theta_{0c}$ do not differentiate too much. This makes it possible to observe the soliton states at a fixed input power for different pretilt angles (or different bias voltages).[16] Figure 2 presents the simulation results of two solitons with a relative angle of 0.57° for different values of $\theta_0$. We can see that for $\theta_0\leqslant\pi/4$, the nonlocality is strong enough to guarantee the attraction of both the in-phase and the out-of-phase solitons. However, the degree of nonlocality becomes lower than the critical degree of nonlocality when $\theta_0=0.45\pi$. In this case, the out-of-phase solitons begin to repulse each other and the in-phase solitons remain attraction.

The experimental setup is illustrated in Fig. 3. The laser beam from an argon-ion laser is split into two beams, then they are combined together with a small separation through the other beam splitter and launched into a 80-$\mu$m-thick NLC cell by a 10× microscope objective. The beam width at the focus $w_0$, the separation $d_s$, and relative angle $\beta$ between the two beams are measured by an edged-scanning beam profiler when the NLC cell is removed. The phase difference between the two beams is adjusted by the rotation of a 1.8-mm-thick parallel-face plate, and measured through the

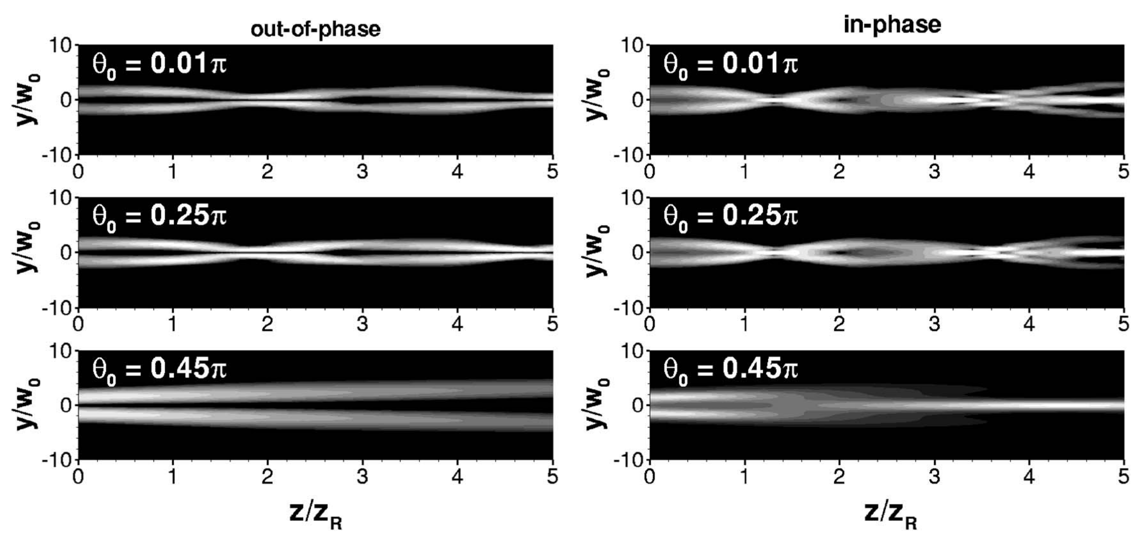


FIG. 2. Numerical simulation results of the interactions of the in-phase and the out-of-phase solitons. The width for each soliton is 4 $\mu$m and the input power is 1.1 mW. The separation and the relative angle between two solitons are, respectively, 12 $\mu$m and 0.57° ($\tan 0.57°=0.01$).

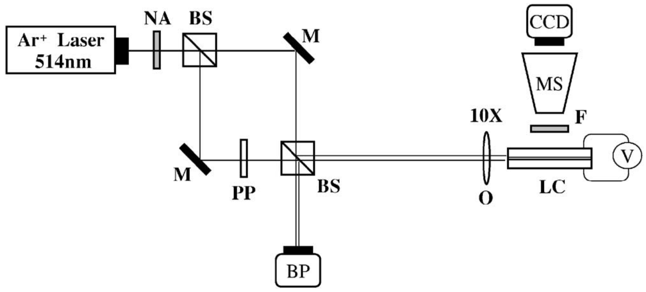


FIG. 3. Scheme of the experimental setup. NA, neutral attenuator; BS, beam splitters; M, plate mirror; PP, parallel-face plate for adjusting the phase difference; O, 10× microscope objective; LC, liquid crystal cell; MS, microscope; F, laser-line filter; and BP, beam profiler.

interference pattern by the beam profiler located on the other branch after the second beam splitter. The cell is filled with the NLC TEB30A (from SLICHEM China Ltd.), whose $n_{\|}=1.6924$, $n_{\perp}=1.5221$, $K\approx 10^{-11}N$, $\epsilon_a^{\rm op}=0.5474$, and $\epsilon_a^{\rm rf}=9.4$. The Freèdericksz threshold $V_t\approx 1.14$ V for the 80-$\mu$m-thick cell. The launched power for each beam is fixed to 7 mW when the bias is changed, and the other parameters for the beams in the NLC are $w_0=3.2$ $\mu$m, $d_s=10$ $\mu$m, and $\tan\beta=0.011$. When the phase difference is adjusted to 0 or $\pi$, we record the beam traces for the different biases by the charge-coupled device camera, as shown in Fig. 4.

Let us compare the photos for the in-phase and the out-of-phase solitons when the bias $V=1.4$ V ($\theta_0\approx\pi/4$). They are almost the same for both cases. It means for $\theta_0=\pi/4$ the degree of nonlocality is strong enough to eliminate the dependence of the interactions on the phase difference between the solitons. In this case, $w_m\approx 25.3$ $\mu$m, which is bigger than the separation of the two beams, and $\alpha=0.126$ for the 3.2-$\mu$m-width solitons.

For the bias $V=1.0$ V slightly lower than the threshold $V_t=1.14$ V (a small tilt angle in the sample educes some reorientation at the bias voltage lower than the threshold[12]), the pretilt angle $\theta_0$ is nearly zero and nonlocality is much stronger than that when $V=1.4$ V. For this reason, a second cross point is observed for both the in-phase and the out-of-phase solitons.

When the bias $V$ (pretilt angle $\theta_0$) increases, the degree of nonlocality $1/\alpha$ and the characteristic length $w_m$ decrease. For $V=2.4$ V ($\theta_0\approx 0.45\pi$), we have $w_m\approx 11$ $\mu$m, which approximately equals to the separation between the two solitons. In this case, we observe the attraction of the in-phase solitons and the repulsion of the out-of-phase solitons. We also see the two in-phase solitons fused into one soliton, which is qualitatively the same with the numerical simulation result in Fig. 2.

In conclusion, we have investigated theoretically and experimentally the interactions of the nonlocal spatial solitons

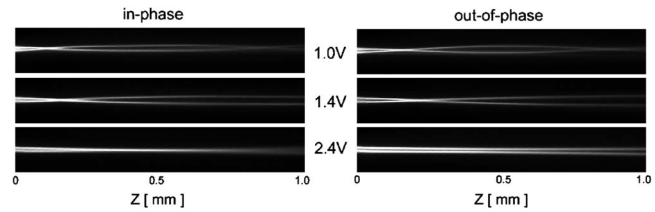


FIG. 4. Photos of the beam traces for the soliton pair propagation in the NLC cell. The biases applied on the LC are 1.0, 1.4, and 2.4 V, corresponding to the pretilt angles of $0.01\pi$, $0.25\pi$, and $0.45\pi$, respectively.

in the NLC when the applied bias is adjusted. It is shown that the voltage-controllable degree of nonlocality in the NLC can be implemented expediently. We experimentally observe the transition from attraction to repulsion of the two out-of-phase solitons in the NLC as the degree of nonlocality decreases via increasing the bias. Such a voltage-controllable soliton interaction might have its potential applications in developing all-optical signal processing devices.

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[16] Rigorously speaking, except one of them, they are quasisolitons (breathers) rather than solitons because the fixed input power can only equal exactly one of the critical powers for different biases, but is close to the others in the case under consideration.