Unidirectional Transmission in Asymmetrically Confined Photonic Crystal Defects with Kerr Nonlinearity *

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We investigate the unidirectional transmission behaviour of an asymmetrically confined photonic crystal (PC) defect with Kerr nonlinearity. Basically, the unidirectional transmission originates from the strong dependence of the threshold input power for the sharp increase of transmission on the launch direction of the input wave. This can be well explained in the framework of the coupled mode theory. Our theoretical analysis reveals the existence of an upper limit for the transmission contrast when such a single PC defect is employed. This is supported by the simulation results based on the nonlinear finite-difference time-domain technique.

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Photonic crystals (PCs) formed by periodic modulation of refractive index become a magic platform, where many devices such as low-loss waveguides, delayed lines, filters, switches, and lasers, etc. are created.[1−6] These PC-based devices hold novel characteristics that satisfy the requirement of optical integration and all-optical communication. Among them, PC-based optical diodes, which transmit light of given frequency and power unidirectionally, have attracted much attention. The first proposal for PC-based optical diodes was suggested by Scolora *et al.*[7,8] based on the dynamical shift of the photonic band edges in a structure that consists of alternate stacks of linear and nonlinear layers. After several years, Gallo *et al.*[9] demonstrated that an all-optical diode can be created by utilizing a $LiNiO_3$ waveguide with gratings in which a phase shift is introduced asymmetrically, causing the transition between the second harmonic wave and the basic wave waveguiding-direction related. Also, Mingaleev *et al.*[10] constructed an all-optical diode in a line-defect waveguide composed of four nonlinear PC defects arranged in an asymmetric fashion. Very recently, Michael *et al.*[11] investigated the bistable diode action in an asymmetric multi-layer structure constructed by alternate layers made of conventional and left handed materials.

It is realized from the previous studies on all-optical diodes that the combination of *asymmetric* structure and nonlinearity in PCs leads to optical devices with unidirectional transmission. In general, the nonlinear processes occurring in PCs are rather complicated. Thus, PC structures with simple configurations possess advantages in revealing the physical mechanisms that govern the complicated phenomena. Keeping this in mind, it is obvious that an asymmetrically confined PC defect can be employed to build an all-optical diode and to manifest the physical mechanism. On the other hand, a PC defect is sometimes considered as the basic element of many structures, so a good understanding of its unidirectional potential seems to be very essential. In this Letter, we focus on a PC defect confined asymmetrically in a slab waveguide that is made of Kerr media. The factors that affect the unidirectional transmission are investigated theoretically and numerically.

The asymmetrically confined PC defect we studied is shown in the inset of Fig. 1. Similar structures have been investigated and fabricated several years ago for optical filters.[1] The PC structure is formed by periodically drilling seven identical air holes in a GaAs slab waveguide. The distance between two neighbouring air holes is $a = 0.45\,\mu$m. One PC defect is introduced by increasing the distance between the third and the fourth air holes to $1.35a$ intentionally. Apparently, the defect created by this way is asymmetrically confined. The width of the slab waveguide and the radius of air holes are chosen to be a and $0.3a$, respectively. Since it has been confirmed that the simulation of a pure two-dimensional (2D) structure is a good approximation for the practical 2D PC slabs,[12] we choose to perform numerical simulations on a 2D nonlinear PC structure with a linear effective refractive index of $n_{\rm eff} = 2.89$. The nonlinear coefficient for the GaAs waveguide n_2 is assumed to be $1.5 \times 10^{-5}\,\mu\text{m}^2/\text{W}$. We focus on the transverse electric (TE) mode of the waveguide whose magnetic field is parallel to the air rods. It possesses a band gap ranging from $0.256(2\pi c/a)$ to $0.327(2\pi c/a)$, where c is the speed of light in vacuum. The transmission spectrum of the PC defect is shown in Fig. 1. It can be seen that the resonant frequency ω_0 is $0.287(2\pi c/a)$, or the wavelength λ_0 is about $1.57\,\mu$m. Its linewidth

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γ is estimated to be 8.54×10^{-4} $(2\pi c/a)$.

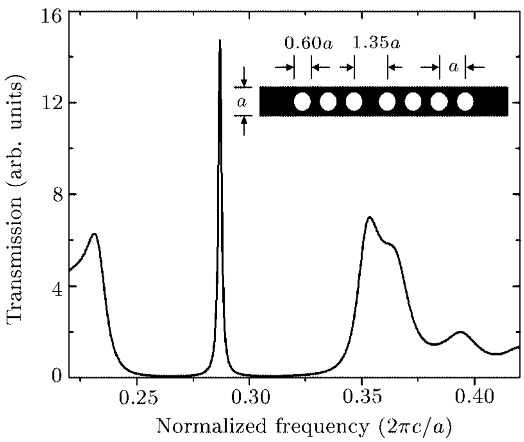

Fig. 1. Linear transmission spectrum of the asymmetrically confined PC defect with Kerr nonlinearity created in a GaAs slab waveguide. The schematic structure is shown in the inset.

Now let us examine the transmission behaviour of the nonlinear and asymmetrically confined PC defect described above. To do so, continuous waves (CWs) with a frequency detuning $\delta = (\omega_0 - \omega)/\gamma = 3.0$ with respect to the defect mode are launched into the GaAs waveguide from the left and right sides, respectively. In both the cases, the stable transmissions that correspond to the different values of input power $p_{\rm in}$ are recorded at the output port by using the nonlinear finite-difference time-domain (FDTD) technique (in this study, a commercial software developed by Rsoft Design Group (http://www.rsoftdesign.com) is used for nonlinear FDTD simulation). The results are presented in Fig. 2. It is found that in both the cases the transmission exhibits very similar dependence on the input power. The remarkable feature is the sharp increase of the transmission when the input power reaches a certain level. We term this level as the threshold for the jump of the transmission. This phenomenon arises from the shift of the defect mode towards the input frequency that forms a positive feedback process. However, it is noticed that the threshold for the transmission jump depends on the launch direction of the input wave. Since the input wave is easier to be coupled into the defect from the left side than from the right one, the corresponding threshold power is found to be much lower. This unique feature provides us an opportunity to construct an all-optical diode by employing such a nonlinear and asymmetrically confined PC defect. It is apparent that the difference between the transmission for the rightward waves $(T_{\rm right})$ and that for the leftward ones $(T_{\rm left})$ becomes obvious when $p_{\rm in}$ exceeds $\sim 15\,{\rm W}/\mu{\rm m}$. For convenience, we use transmission contrast C, which is defined as $C = T_{\rm righ}/T_{\rm left}$, to characterize the unidirectional transmission behaviour of the PC defect. Obviously, its value is larger than one for $p_{\rm in} < 38\,{\rm W}/\mu{\rm m}$ and it becomes smaller than one when $p_{\rm in} > 38\,{\rm W}/\mu{\rm m}$. The maximum transmission contrast $C_{\rm max}$ is achieved at $p_{\rm in} = 20\,{\rm W}/\mu{\rm m}$ where the transmission jump for the rightward wave occurs. $C_{\rm max}$ is estimated to be 5.6 which is not so large. A similar phenomenon has been addressed very recently in Ref. [11], where a left-handed periodic structure was adopted and simulated by using the transfer matrix method.

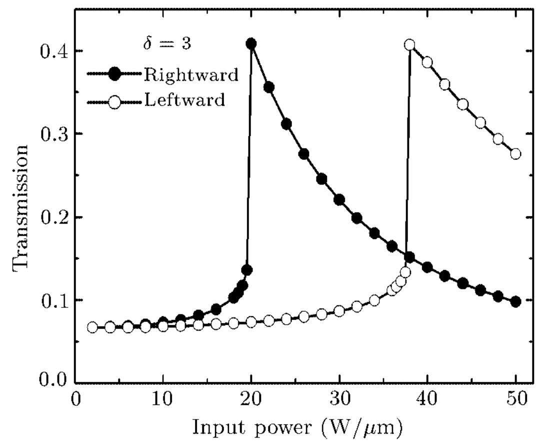

Fig. 2. Transmission for the two incident directions as a function of the input power.

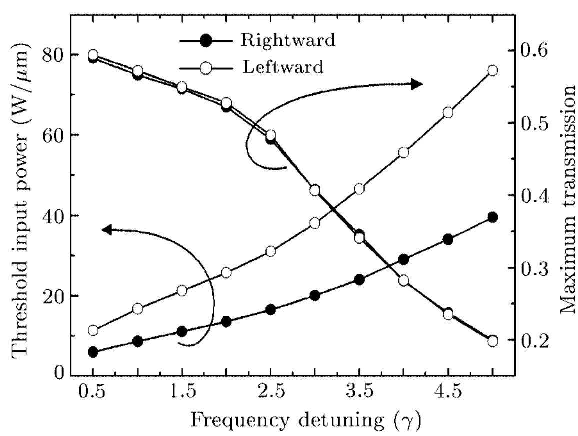

Fig. 3. Dependences of the thresholds for the transmission jump and the maximum transmission on the frequency detuning of the input wave.

It becomes clear that unidirectional transmission observed in the nonlinear asymmetrically confined PC defect originates from the dependence of the threshold input power for the transmission jump on the launch direction. In principle, this phenomenon can be found for incident waves with any frequency detunings. The thresholds for the rightward and leftward waves as a function of the frequency detuning are shown in Fig. 3. The evolutions of the maximum values for $T_{\rm righ}$ and $T_{\rm left}$ with increasing the frequency detuning are also presented. It can be seen that the two thresholds and the difference between them increase monotonically with increasing δ. Meanwhile, the maximum values for $T_{\rm righ}$ and $T_{\rm left}$ depend only on the frequency detuning and they have nothing to do with the launch direction. Obviously, the maximum values for $T_{\rm righ}$ and $T_{\rm left}$ exhibit monotonic decreases with increasing d. These phenomena can be easily understood by use of the dynamical shift model of the defect mode. Actually, we have indicated that the defect mode oscillates harmonically under the excitation of an external pump and the observed transmission is the time-averaged value within an oscillation period.[13] Thus, a larger d results in a smaller value for the maximum transmission. The launch direction only affects the coupling of the input wave into the defect and thus the threshold. It has no effects on the maximum transmission.

Theoretically, this issue can be well explained in the framework of the coupled mode theory (CMT).[14] Previously, the CMT has been successfully used by Soljacic *et al.*[17] to describe the optical bistability of a nonlinear PC defect. If we denote the field amplitude of the input wave entering into the PC defect and that of the output wave emerging from the PC defect as $s_{\rm in}$ and $s_{\rm out}$, then, according to the CMT, the equation describing the relationship between $s_{\rm out}$ and $s_{\rm in}$ can be written as

$$\frac{ds_{\rm out}}{dt} = \Big[j\Big(\omega_0 - \gamma\frac{|s_{\rm out}|^2}{p_0}\Big) - \gamma\Big]s_{\rm out} + \sqrt{\frac{4}{\tau_R\tau_L}}s_{\rm in}, \quad (1)$$

where p_0 is the characteristic power that reflects the nonlinear Kerr effect and the spatial confinement of the field in the defect region,[15] $(\gamma|s_{\rm out}|^2/p_0)$ represents the shift of defect mode frequency because of the nonlinear Kerr effect,[15] γ is the total decay rate of the mode amplitude, $1/\tau_R$ and $1/\tau_L$ denote the rightward and leftward decay rates of the mode amplitude, respectively. By using the steady solution of Eq. (1), the stable transmission through the PC defect can be derived to be

$$T = \Big|\frac{s_{\rm out}}{s_{\rm in}}\Big|^2 = \frac{p_{\rm out}}{p_{\rm in}} = \frac{\eta}{1+(p_{\rm out}/p_0-\delta)^2}, \quad (2)$$

where $\eta = 4/(\gamma^2\tau_R\tau_L)$ is the resonant power transmission of the PC defect in the linear case and $p_{\rm out}$ denotes the stable output power. In the cases of $\delta > \sqrt{3}$, it is not difficult to derive the stable transmission based on Eq. (2) that a sharp increase of transmission from $T_{\rm down}$ to $T_{\rm up}$ occurs when the input power $p_{\rm in}$ reaches a threshold given by

$$p_T = \frac{2\delta(\delta^2+9)+2(\delta^2-3)^{3/2}}{27\eta}p_0. \quad (3a)$$

In addition, the transmissions before ($T_{\rm down}$) and after ($T_{\rm up}$) the transition can be expressed as

$$T_{\rm down} = 4.5\eta(2\delta-\sqrt{\delta^2-3})/[\delta(\delta^2+9)+(\delta^2-3)^{3/2}], \quad (3b)$$

$$T_{\rm up} = 9\eta(\delta+\sqrt{\delta^2-3})/[\delta(\delta^2+9)+(\delta^2-3)^{3/2}]. \quad (3c)$$

When the optical bistability of nonlinear PC defects is concerned, this process is generally referred to as the jump from the lower branch of the hysteresis to the upper one. Since the characteristic power p_0 depends on the coupling of the input wave with the defect, we use p_{01} and p_{02} to present the characteristic powers for the rightward and the leftward waves respectively. To do so, the phenomena described above can be well explained. For example, it is obvious from Eq. (3a) that the direction-related threshold p_T increases monotonously with the increase of δ, and the difference between the two thresholds for the two launch directions also increases when increasing δ. In addition, it is easy to find that $T_{\rm up}$ actually represents the maximum transmission as shown in Fig. 2. According to Eq. (3b), it is found that $T_{\rm up}$ is the function of η and δ and it has nothing to do with p_0 (or the launch direction). Therefore, the results derived from the CMT are in good agreement with the simulation results presented in Fig. 3. In the cases of $\delta \le \sqrt{3}$, the transmission reaches its maximum value η when $p_{\rm in} = \delta p_0/\eta$ and it exhibits the unidirectional behaviour to some extent. However, the maximum transmission contrast $C_{\rm max}$ is much smaller as compared to the cases of $\delta > \sqrt{3}$ because no jump of transmission occurs as the input power is increased.

It has been shown that the value of $C_{\rm max}$ is not so large for a single asymmetrically confined PC defect with Kerr nonlinearity. Theoretically, we can derive the upper limit of $C_{\rm max}$ based on the equations presented above. Now let us examine the cases of $\delta > \sqrt{3}$ and $C > 1$. In this case, the transmission for the leftward wave can be approximated to be

$$T_{\rm left} \cong \frac{\eta}{1+\delta^2}\Big[1+\frac{2\delta}{(1+\delta^2)^2}\cdot\frac{p_{\rm in}}{p_{02}}\Big], \quad (4)$$

by using an iterative operation on Eq. (2). Thus, the lower limit of $T_{\rm left}$ is $\eta/(1+\delta^2)$. Apparently, $C_{\rm max}$ is achieved when transmission jump for the rightward wave occurs, i.e., when $p_{\rm in}$ reaches the threshold and $T_{\rm right}$ equals $T_{\rm up}$. Therefore, we can obtain the following expression:

$$C_{\rm max} = \frac{T_{\rm right}}{T_{\rm left}} \le 9(\delta^2+1)(\delta+\sqrt{\delta^2-3})/[\delta(\delta^2+9) + (\delta^2-3)^{3/2}]. \quad (5)$$

This expression indicates that there always exists an upper limit for $C_{\rm max}$ when δ is decided. When δ approaches infinity, the value of the right part of Eq. (5) approaches 9, which is considered to be the upper limit of $C_{\rm max}$ for a single asymmetrically confined PC defect with Kerr nonlinearity. It sets a serious limit for the optical diodes constructed with a single PC defect. In Fig. 4, we present a comparison of the upper limit of $C_{\rm max}$ obtained by Eq. (5) with $C_{\rm max}$ extracted from the FDTD simulations. They exhibit similar saturated dependence on the increase of frequency detuning. The absolute values are different because the right part of Eq. (5) is the upper limit of $C_{\rm max}$, not $C_{\rm max}$ itself.

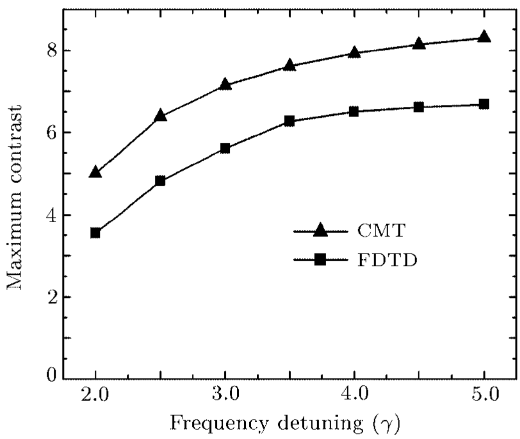

Fig. 4. Comparison of the upper limit for the maximum contrast derived by the CMT with the maximum contrast obtained by FDTD simulation.

At first glance, the value of $C_{\max}$ may be enhanced by making the confinement of the PC defect more asymmetrical. However, a detailed analysis reveals that it leads only to a larger characteristic power p_{02}, making $T_{\rm left}$ closer to its lower limit $\eta/(1+\delta^2)$ and $C_{\max}$ closer to its upper limit. Thus, it is definitely not an effective way to improve $C_{\max}$. On the other hand, increasing the asymmetry of the PC defect will result in a marked drop of the maximum transmission as the peak transmission in the linear case η becomes very small. Therefore, a good unidirectional behaviour cannot be achieved by only increasing the asymmetry of the PC defect although it is necessary to realize an optical diode.

From the above discussion we conclude that the unidirectional transmission of an asymmetrically confined PC defect with Kerr nonlinearity can be investigated in the framework of the CMT. The theoretical analysis provides us some useful information for the unidirectional transmission behaviours such as the upper limit of $C_{\max}$ derived above.

Before drawing conclusions, let us see the transmission of a short pulse through the nonlinear PC defect which is more practical than the CW case. The pulse width and the frequency detuning are chosen to be 1.56 ps and 3, respectively. Similar to the CW case, we measure the output signals corresponding to the two launch directions and we define $C_{\max}$ as the maximum ratio of the peak powers of the two output pulses. Figure 5 shows the peak transmission for the two launch directions with respect to the input peak power. A transmission jump is observed when the peak power of the input pulse increases to about 23 W/μm for the rightward wave and about 45 W/μm for the leftward one. Thus, the unidirectional transmission occurs when the peak power of the input wave is within the range of 15 W/μm and 40 W/μm. In this case, $C_{\max}$ is calculated to be nearly 4.6, which is smaller than the value in the CW case (about 5.6). Nevertheless, it is confirmed that the unidirectional transmission for the pulse case can also be explained by the CMT. The inset of Fig. 5 shows the output pulses for two launch directions when the peak power of the input pulse is chosen to be about 23 W/μm. A slight distortion in pulse shape and a slight narrowing in pulse width are observed for the rightward pulse. In comparison, the distortion for the leftward pulse is negligible while a slight broadening in pulse width is found. These phenomena can also be explained by the dynamical shift of the defect mode.[16]

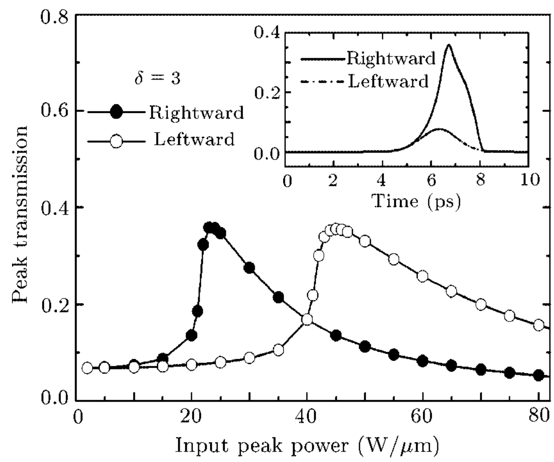

Fig. 5. Dependence of the peak transmission for the two incident directions as a function of the peak power of the input pulse. Output pulses at an input power of 23 W/μm are shown in the inset.

In summary, we have investigated theoretically and numerically the unidirectional transmission behaviour of a single asymmetrically confined PC defect with Kerr nonlinearity. It is revealed that the dependence of the characteristics power on the launch direction is responsible for the observed unidirectional transmission. In addition, we have derived the upper limit of the transmission contrast and indicated the limitation of using a single nonlinear PC defect for realizing optical diodes. Theoretical analyses are in good agreement with the simulation results.

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