Physical origin of the ultrafast response of nonlinear photonic crystal atoms to the excitation of ultrashort pulses

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The physical origin of the ultrafast response of nonlinear photonic crystal (PC) atoms to the excitation of ultrashort pulses is investigated in detail. It is found that the dynamic shift of PC defect modes is responsible for the dramatic shortening of the photon lifetime in nonlinear PC atoms. The transmission spectra for nonlinear PC atoms that resulted from the dynamic shift are calculated and they are in good agreement with the simulation results.

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I. INTRODUCTION

Photonic crystals (PCs) formed by periodic modulation of dielectric constants act as a promising platform to manipulate photons.[1] By intentionally introducing defects into perfect PCs, various functional devices have been proposed and demonstrated.[2–7] As the simplest version of PC defects with various applications, a point defect which is sometimes referred to as a PC atom has been extensively investigated. One of its potential applications is the construction of high-efficiency all-optical switches.[6,7] The proposal of utilizing a single PC atom with nonlinearity to build an all-optical switch was first suggested by Villeneuve *et al.*[6] The switching mechanism is based on the shift of the PC defect mode as the refractive index in the defect region is modified by an external pump. This model provides a basic idea for realizing all-optical switches. It has been indicated that the switching speed is ultimately limited by the photon lifetime in the PC atom. However, the interaction of nonlinear PC atoms with external pump (usually ultrashort pulses) has not yet been studied. This issue is not only important for fundamental physics but also attractive for device applications.

Very recently, we carried out an initial study on the response of PC atoms with Kerr nonlinearity to the excitation of ultrashort pulses.[8] We revealed that PC atoms with sharp resonant peaks (or large quality factors) could respond to ultrashort pulses very rapidly when Kerr nonlinearity was introduced. It is found that the significant broadening of the frequency spectra of nonlinear PC atoms is responsible for the ultrafast response time.[8] Therefore, it is necessary to find out the physical origin and major features of the spectral broadening of nonlinear PC atoms under high excitation densities. In this paper, we present a detailed analysis for the characteristics of PC atoms with Kerr nonlinearity under the excitation of continuous and pulse sources, trying to provide a physical insight into the dynamics of nonlinear PC atoms.

II. PHYSICAL MODEL FOR THE DYNAMICS OF NONLINEAR PC ATOMS

For a PC atom with Kerr nonlinearity, its frequency spectrum (or the defect mode) will be shifted instantaneously once the PC atom is excited by an external pump. It is well known that the shift of the defect mode depends on the intensity of the external pump. However, very little attention has been paid to the dynamic feature of the shift which is crucial for the physical properties of nonlinear PC atoms. The dynamic shift means the shift repeats in a frequency that is the same as the pump wave. Thus, the first issue one needs to clarify is the effects of the dynamic shift of a PC defect mode on the transmissions observed at frequencies near the defect mode. Based on this, the physical model for the dynamics of nonlinear PC atoms can be established.

Owing to the dynamic shift, the transmission observed in the steady state is actually the time average of the transient values which depend on the locations of the defect mode. As an example, we investigate for simplicity a two-dimensional (2D) PC atom with Kerr nonlinearity. Its structure is depicted in Fig. 1(a). The 2D PC consists of a square lattice (7×5) of GaAs dielectric rods with a linear refractive index of $n_0=3.4$. Their radius is chosen to be $0.2a$, where $a=0.6$ μm is the lattice constant of the PC. The nonlinear PC atom is formed by decreasing the radius of the central rod to $0.1a$ and introducing Kerr nonlinearity into it. It means that the refractive index at any point of the defect $n(x,z)$ is proportional to the local electric field intensity $E^2(x,z)$, i.e., $n(x,z)=n_0+n_2E^2(x,z)$, where n_2 is the nonlinear coefficient. The first band gap of the 2D PC for the transverse magnetic polarization ranges from $0.2750(2\pi c/a)$ to $0.4307(2\pi c/a)$ and the defect mode is located at $0.3282(2\pi c/a)$ in the absence of an external pump.

Now we consider the transmission of the nonlinear PC atom upon the excitation of continuous waves (CW) whose frequencies are close to that of the defect mode. The linear frequency spectrum of the PC atom and the pump schemes are shown in Fig. 1(b). The nonlinear coefficient for the PC defect is assumed to be $n_2=1\times10^{-5}$ μm^2/W (or 1×10^{-13} cm^2/W) which is very close to the practical values of GaAs and AlGaAs ($\sim2\times10^{-13}$ cm^2/W).[9] Accordingly, the three pump waves are set at the low-frequency (or long-wavelength) side of the defect mode and their frequencies (or wavelengths) are chosen to be $\omega_1=0.3279(2\pi c/a)$ (or $\lambda_1=1.83$ μm), $\omega_2=0.3261(2\pi c/a)$ (or $\lambda_2=1.84$ μm), and $\omega_3=0.3243(2\pi c/a)$ (or $\lambda_3=1.85$ μm). The nonlinear finite-difference time-domain (FDTD) technique is employed to simulate the transmission behavior of the nonlinear PC atom

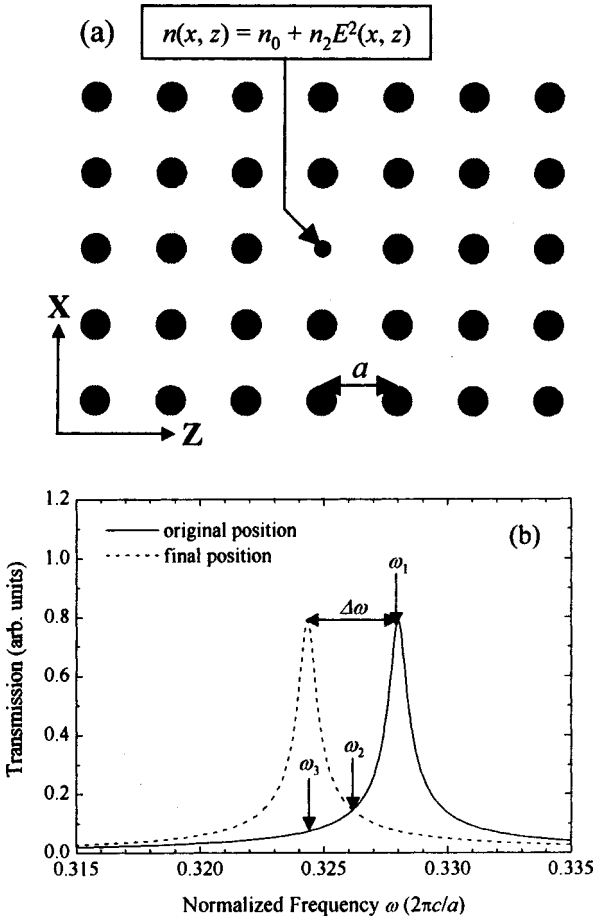

FIG. 1. (a) Structure of the nonlinear PC atom. (b)Linear transmission spectrum of the PC atom and the three pump schemes in case of CW excitation. The dashed curve shows the final position of the defect mode after a frequency shift of $\Delta\omega$.

under the excitation of CW waves.[10] The grid size used in the simulation is $a/20$ for both directions. Further reduction in grid size barely influences the simulation results. The dependencies of the transmission on the pump intensity obtained by FDTD simulation for the three pump schemes are presented in Fig. 2. In all cases, it can be seen that the transmission increases slowly at low pump intensities. For $\omega_1 = 0.3279(2\pi c/a)$, this process occurs at very low excitation densities and it is not obvious in Fig. 2. However, a sharp increase in transmission is observed as the pump intensity reaches a threshold. After that, the transmission starts to decrease gradually. The threshold at which the rapid increase of transmission occurs strongly depends on the pump frequency (wavelength). Another important feature we noticed is that the maximum transmission achieved also strongly depends on the pump frequency (wavelength). For pump frequencies (wavelengths) far from the defect mode [i.e., $\omega_3 = 0.3243(2\pi c/a)$ or $\lambda_3 = 1.85\ \mu$m], its value is much smaller than the peak transmission of the defect mode. As the pump frequency (wavelength) becomes close to the defect mode, its value approaches the peak transmission of the defect mode. In the steady state, the PC defect mode oscillates harmonically between its initial position and a final position which is dependent on the intensity of the pump source. The

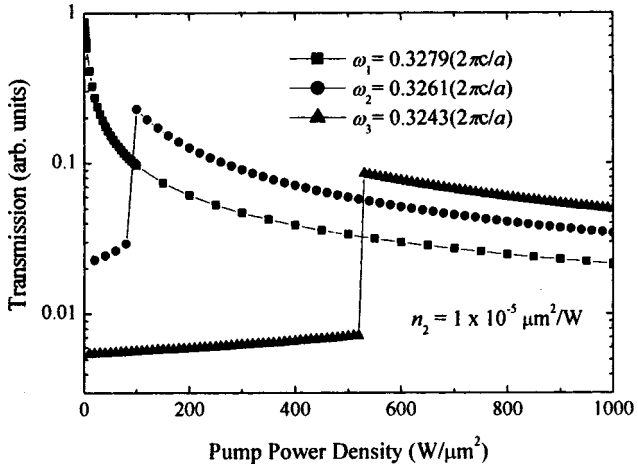

FIG. 2. Dependencies of the transmission of the nonlinear PC atom on the pump intensity for the three pump schemes obtained by FDTD simulation.

oscillation frequency is the same as that of the pump wave. Therefore, the transmission observed at the pump frequency (wavelength) is actually the time average of the transient values within an oscillation period. Such a physical picture is supported by the second feature of the nonlinear PC atom described above. Since the transmission we observed in the simulation is the time-averaged transmission of the transient values within an oscillation period, it is always smaller than the peak transmission of the defect mode, especially for the case when the pump frequency (wavelength) is far from the defect mode.

In order to confirm the validity of this physical model, we have calculated the time-averaged transmissions as a function of defect mode shift for the three pump schemes. They are presented in Fig. 3. The time-averaged transmission within a period is given by the integral of the defect mode between its initial and final positions divided by the shift of the defect mode, that is

$$\overline{T}(\omega) = \frac{1}{\Delta\omega}\int_{\omega_{\text{pump}}}^{\omega_{\text{pump}}+\Delta\omega} T(\omega), \tag{1a}$$

or equivalently

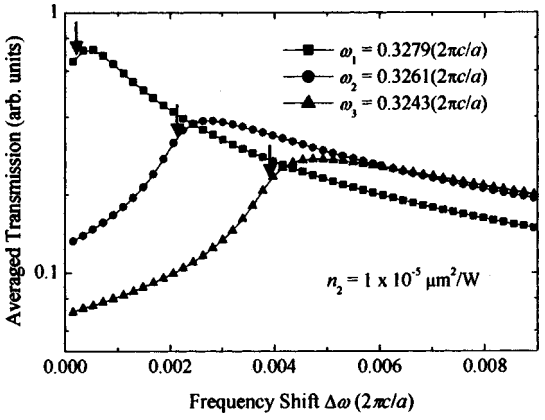

FIG. 3. Time-averaged transmissions as a function of the defect mode shift for the three pump schemes.

$$\overline{T}(\lambda) = \frac{1}{\Delta\lambda}\int_{\lambda_{pump}-\Delta\lambda}^{\lambda_{pump}} T(\lambda). \tag{1b}$$

Here, $T(\omega)$ or $T(\lambda)$ is the linear transmission spectrum of the PC atom, ω_{pump} (λ_{pump}) is the frequency (wavelength) of the pump wave, and $\Delta\omega$ ($\Delta\lambda$) is the induced frequency (wavelength) shift of the defect mode. For simplicity, it has been assumed that the defect mode moves uniformly between its initial and final positions. We can see later that this is not true for the moving of the defect mode. Obviously, the calculated results look closely to the simulated one, justifying the validity of our physical model for the dynamics of nonlinear PC atoms. However, the increase in transmission obtained in the calculated results is not as sharp as that observed in the simulated one. This discrepancy can be attributed to the nonlinear relationship between the shift of the defect mode and the pump intensity. It has been indicated in Ref. 11 that a positive feedback is present as the defect mode shifts towards the pump frequency (wavelength). It is responsible for the sharp increase in transmission observed in the simulation results. From the calculated results, it is also clarified that the coincidence of the defect mode with the pump wave only corresponds to the sharp increase in the transmission, not to the maximum transmission. In Fig. 3, we indicated by arrows the points at which the peak of the defect mode is shifted to the pump frequency (wavelength).

III. TRANSMISSION SPECTRA OF NONLINEAR PC ATOMS: SIMPLIFIED MODEL

Based on the physical model for the dynamics of nonlinear PC atoms established above, we can easily obtain the transmission spectrum of a nonlinear PC atom upon the excitation of a CW source. In general, PC defect modes possess Lorentz lineshapes.[12] In the case when the PC defect mode has a lineshape of another type, the procedure for the derivation of the transmission spectra of nonlinear PC atoms described in the following is still applied but the analytical expression for the transmission spectra may be different. In frequency spectrum, the defect mode of a PC atom can be expressed as follows:

$$T(\omega) = \frac{\gamma^2}{\gamma^2 + (\omega - \omega_0)^2}, \tag{2}$$

where $T(\omega)$ is the linear transmission spectrum of the PC atom, ω_0 is the resonant frequency of the defect mode and γ is the half of the linewidth of the defect mode. Under the excitation of a CW source, the frequency shift of the defect mode $\Delta\omega$ is a constant that gives the final location of the defect mode ($\omega_0 - \Delta\omega$). The sign of $\Delta\omega$ is determined by that of n_2. If we assume that the defect mode moves uniformly between its initial and final positions, then the time-averaged transmission $T(\omega)$ within a period is given by

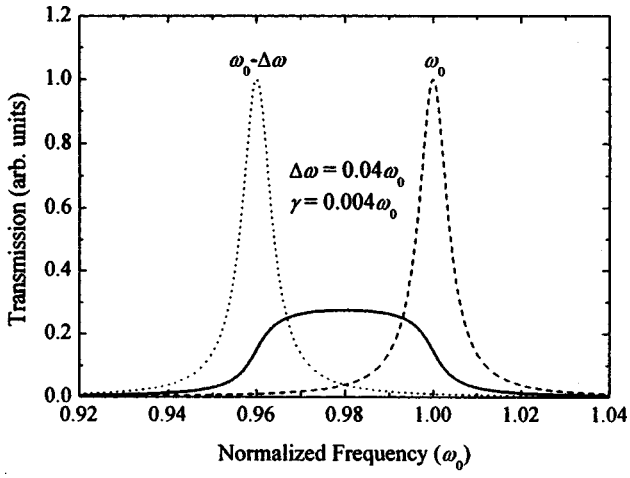

FIG. 4. Transmission spectrum of a nonlinear PC atom driven by a CW source (solid curve) based on the simplified model ($\Delta\omega = 0.04\omega_0$ and $\gamma = 0.004\omega_0$). The spectra of the defect mode at the initial and final positions are shown by the dashed and dotted curves, respectively.

$$\overline{T}(\omega) = \frac{1}{\Delta\omega}\int_{\omega}^{\omega+\Delta\omega} \frac{\gamma^2}{\gamma^2 + (\omega - \omega_0)^2} d\omega. \tag{3}$$

This expression describes the transmission spectrum of a nonlinear PC atom which is driven by a CW source. After a simple calculation, we obtain

$$\overline{T}(\omega) = \frac{\gamma}{\Delta\omega}\left[\arctan\left(\frac{\omega + \Delta\omega - \omega_0}{\gamma}\right) - \arctan\left(\frac{\omega - \omega_0}{\gamma}\right)\right]. \tag{4}$$

The transmission spectrum of the nonlinear PC atom described by Eq. (4) is plotted in Fig. 4 ($\Delta\omega = 0.04\omega_0$ and $\gamma = 0.004\omega_0$). It can be seen clearly that the dynamic shift of the defect mode indeed leads to a marked broadening of the spectral linewidth. In addition, a shift of the transmission peak to low frequency is observed. The larger the shift of the defect mode, the wider the broadening of the spectrum is. Therefore, we have used a simplified model to show that the dynamic shift of the defect mode does introduce a pronounced broadening in its frequency spectrum. This broadening is responsible for the shortening of the photon lifetime in nonlinear PC atoms.

IV. TRANSMISSION SPECTRA OF NONLINEAR PC ATOMS: MORE RIGOROUS MODEL

Although we have demonstrated that the broadening of frequency spectrum is one of the basic features of nonlinear PC atoms, it is obvious that the broadened spectrum shown in Fig. 4 does not look like the one obtained previously by FDTD simulations.[8] Apart from the difference in excitation source (CW or pulse), one reason is that we have made several assumptions in the simplified model. One of them is the uniform moving of the defect mode between its initial and final positions. In fact, the defect mode moves harmonically (i.e., in the form of $\sin \omega t$ and $\cos \omega t$) instead of uniformly

between its initial and final positions when the nonlinear PC atom is driven by an external pump. It means that the velocity of the defect mode is zero at the starting and ending points and it reaches a maximum value in the middle of the moving path. In other words, the defect mode stays at the two ends for a relatively longer time in each period. Therefore, the transmission at both ends is underestimated while that in the middle is overestimated in the simplified model. We can easily imagine the spectral shape of the PC atom in which the transmissions at both ends are enhanced. Mathematically, we can calculate the broadened spectrum of a nonlinear PC atom moving harmonically. The time-averaged transmission within a period can be written as

$$\overline{T}(\omega) = \frac{2}{\pi}\int_{\pi/2}^{\pi} \frac{\gamma^2}{\gamma^2 + (\omega_0 + \Delta\omega \cos^2\theta - \omega)^2} d\theta, \qquad (5)$$

where θ is the phase of the oscillation of the defect mode.

In order to perform a numerical calculation, $T(\omega)$ is converted to the following form:

$$\overline{T}(\omega) = \frac{1}{\pi}\int_{0}^{\pi} \frac{1}{1 + \left(\frac{2\omega_0 + \Delta\omega}{2} + \frac{\Delta\omega}{2\gamma}\cos\varphi - \frac{\omega}{\gamma}\right)^2} d\varphi. \qquad (6)$$

It can be further simplified as follows:

$$\overline{T}(\omega) = \frac{1}{\pi}\int_{0}^{\pi} \frac{1}{1 + (a + b\cos\varphi - c)^2} d\varphi, \qquad \begin{aligned} a &= \frac{2\omega_0 + \Delta\omega}{2\gamma}, \\ b &= \frac{\Delta\omega}{2\gamma}, \\ c &= \frac{\omega}{\gamma}. \end{aligned} \qquad (7)$$

According to Eq. (7), we can easily obtain the time-averaged transmission spectrum of a nonlinear PC atom driven by a CW source within one period. Some examples ($\Delta\omega = 0.04\omega_0$ and $\gamma = 0.004\omega_0$, $0.008\omega_0$, $0.012\omega_0$, $0.016\omega_0$, $0.020\omega_0$) are shown in Fig. 5. When the frequency shift is smaller than or comparable to the linewidth of the defect mode, no big difference can be found between the spectrum obtained by the rigorous model and that calculated by the simplified one. With the increase of the frequency shift, however, the spectral shape is changed. As expected, the transmission at both ends is enhanced while its value in the middle is reduced. The spectral linewidth that reflects the photon lifetime in the nonlinear PC atom is still proportional to the shift of defect mode.

So far, we have achieved the time-averaged transmission spectra for nonlinear PC atoms driven by CW sources. In this case, the shift of the corresponding defect mode $\Delta\omega$ is a constant. Thus the integral given by Eq. (5) can be easily calculated. In the case when a pulse excitation is employed, the situation becomes more complicated because the shift of the defect mode varies with time. In general, the field intensity within the PC atom experiences rise and decay processes. Correspondingly, the frequency shift (or the oscillation amplitude of the defect mode) in Eq. (5) also exhibits

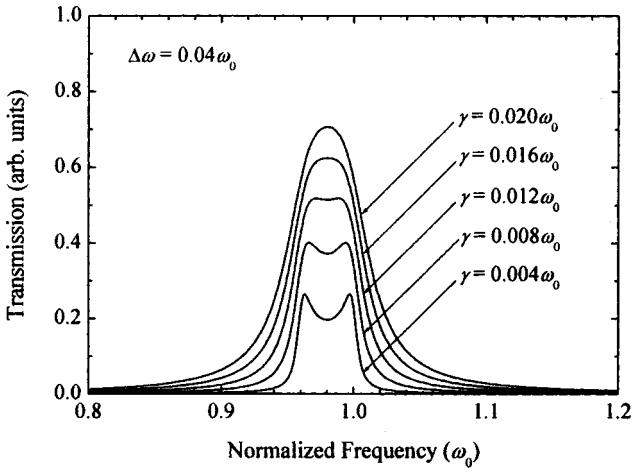

FIG. 5. Transmission spectra of nonlinear PC atoms with a fixed frequency shift ($\Delta\omega = 0.04\omega_0$) and different linewidths ($\gamma = 0.004\omega_0$, $0.008\omega_0$, $0.012\omega_0$, $0.016\omega_0$, and $0.020\omega_0$) driven by a CW source based on the rigorous model.

increase and decrease processes. It should be noticed, however, that the initial position for the defect mode remains unchanged. In Fig. 6, we present the time-averaged frequency spectra within one period for different frequency shifts corresponding to different excitation densities. It can be seen that the single-peak spectra at low excitation densities evolve into spectra with double peaks upon the increase of the excitation density. Meanwhile, it is noticed that the spectral linewidth is markedly broadened and the maximum transmission is accordingly reduced.

In principle, the analytical expression for the transmission spectra of nonlinear PC atoms under the excitation of pulse sources can be obtained by considering the frequency shift involved in Eq. (5) as a function of time. However, it is rather difficult to solve this problem even numerically. An alternative to find the solution is to decompose a transient process into a number of static processes. If we have known the spectrum of each constitutional static process, the spectrum of the transient process can be derived through the su-

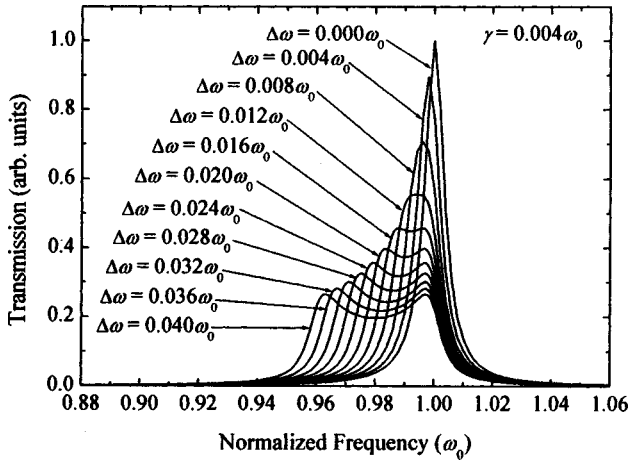

FIG. 6. Transmission spectra of a nonlinear PC atom with a fixed linewidth ($\gamma = 0.004\omega_0$) and different frequency shifts (or under different excitation densities).

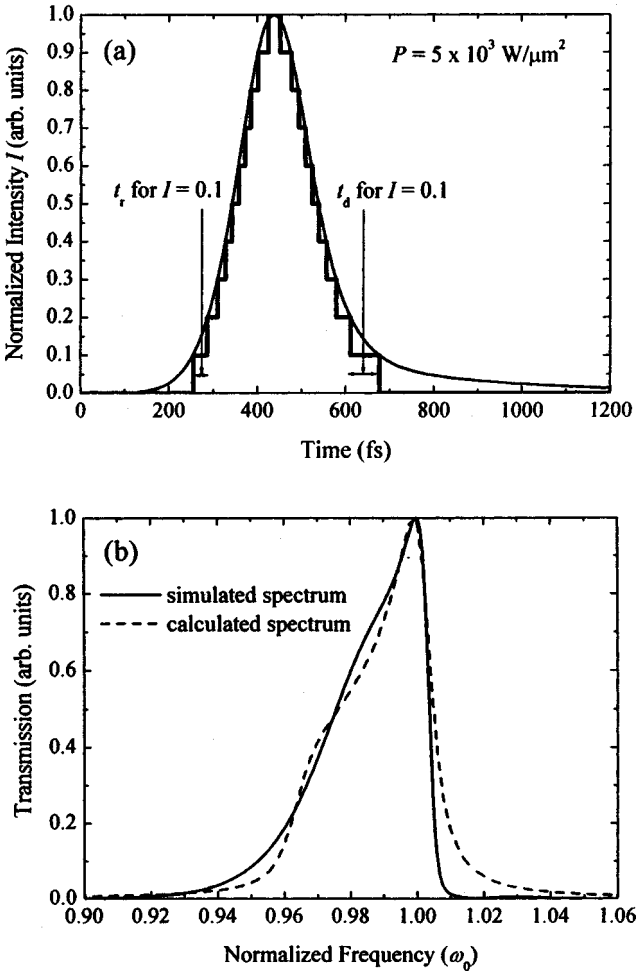

FIG. 7. (a) Evolution of the normalized field intensity in the nonlinear PC atom investigated in Ref. 8 for a pump power density of $P=5\times10^3$ W/μm^2 (thin solid curve). The thick solid curve shows the step function used to approximate the evolution of the field intensity. The duration times of the PC defect mode at a normalized field intensity of $I=0.1$ in the rise (t_r) and decay (t_d) processes are illustrated. (b) Transmission spectra of the nonlinear PC atom obtained by FDTD simulation (solid curve) and calculated by our theoretical model (dashed curve).

perposition of the spectra of all static processes with appropriate weights. To explain the procedure more clearly, we have plotted in Fig. 7(a) the evolution of the normalized field intensity in the nonlinear PC atom studied in Ref. 8 for an excitation density of $P=5\times10^3$ W/μm^2. The approximation of the time evolution of the field intensity with a step function is also presented. For both the rise and decay processes, 11 steps with a constant difference in the field intensity ($\Delta I=0.1$) have been used. According to the analysis given above, it is obvious that the field intensity at each step uniquely determines the shift of the defect mode and thus the transient frequency spectrum of the PC atom. Thus, the 11 steps correspond to the situations of CW excitation with different excitation densities or equivalently with different frequency shifts, as shown in Fig. 6. Therefore, the transmission spectrum can be obtained by the superposition of the 11 spectra given in Fig. 6 with appropriate weights. Apparently, the contribution of each spectrum in Fig. 6 to the total transmission is proportional to the product of the field intensity and the total duration time in the rise and decay processes, i.e., $I(t_r+t_d)$. Thus, we can use $I(t_r+t_d)$ as the weights for the superposition of the 11 spectra. The final result of the superposition is shown in Fig. 7(b). The transmission spectrum obtained by FDTD simulation is also presented for comparison. Since we are mainly concerned with the spectral shape and not the absolute transmittance, the transmission peaks in both spectra have been normalized to one. In addition, the linewidths of the two spectra have been made to be similar in order to ensure that the maximum frequency shifts in the calculation and simulation are almost the same. It can be seen in Fig. 7(b) that very good agreement in spectral shape has been achieved between the simulated and calculated results except for some details. It indicates the validity and consistency of our physical model used to describe the dynamics of nonlinear PC atoms. Obviously, further improvement can be achieved by using many more steps to approximate the evolution of the field intensity.

V. CONCLUSION

In summary, we have investigated in detail the dynamics of nonlinear PC atoms under the excitation of CW and pulse sources. It is found that the dynamical shift of the PC defect modes indeed results in a significant broadening of the transmission spectra for nonlinear PC atoms and leads to a dramatic shortening of the inside photon lifetime. The transmission spectra of the nonlinear PC atoms have been derived analytically and very good agreement between the calculated and simulated results has been achieved.

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[1] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, NJ, 1995).

[2] J. S. Foresi, P. R. Villeneuve, J. Ferrera, E. R. Thoen, G. Steinmeyer, S. Fan, J. D. Joannopoulos, L. C. Kimerling, H. I. Smith, and E. P. Ippen, Nature (London) **390**, 143 (1997).

[3] O. Painter, R. K. Lee, A. Scherer, A. Yariv, J. D. O'Brien, P. D.

Dapkus, and I. Kim, Science **284**, 1819 (1999).
[4] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and E. F. Schubert, Phys. Rev. Lett. **78**, 3294 (1997).
[5] S. Noda, A. Chutinan, and M. Imada, Nature (London) **407**, 608 (2000).
[6] P. R. Villeneuve, D. S. Abrams, S. Fan, and J. D. Joannopoulos, Opt. Lett. **21**, 2017 (1996).
[7] P. Tran, J. Opt. Soc. Am. B **14**, 2589 (1997).
[8] S. Lan, A. V. Gopal, K. Kanamoto, and H. Ishikawa, Appl. Phys. Lett. **84**, 5124 (2004).
[9] J. S. Aitchison, D. C. Hutchings, J. U. Kang, G. I. Stegeman, and A. Villeneuve, IEEE J. Quantum Electron. **33**, 341 (1997).
[10] K. S. Yee, IEEE Trans. Antennas Propag. **AP-14**, 302 (1966). In this article, a commercially available software developed by Rsoft Design Group (http://www.rsoftdesign.com) is used for nonlinear FDTD simulation.
[11] M. Soljačić, M. Ibanescu, S. G. Johnson, Y. Fink, and J. D. Joannopoulos, Phys. Rev. E **66**, 055 601 (2002).
[12] See, for example, Ref. 11; M. F. Yanik, S. Fan, and M. Soljačić, Appl. Phys. Lett. **83**, 2739 (2003); M. Soljačić, C. Luo, J. D. Joannopoulos, and S. Fan, Opt. Lett. **28**, 637 (2003).