Tolerance of Photonic Crystal Impurity Bands to Disorder of Defects in Coupled Cavity Waveguides *

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We investigate the tolerance of photonic crystal impurity bands to the disorder of defects in one-dimensional coupled cavity waveguides. Although impurity bands formed by defect modes close to the air band are quasiflat in the absence of disorder, they are easily deteriorated when disorders are introduced into defects. In contrast, impurity bands created by defect modes near the dielectric band are less sensitive to disorder in the defect size. It is found that the sensitivity of defect mode frequency to defect size and the quality factor of defect modes are two crucial factors in determining the tolerance of impurity bands to the disorder of defects.

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Similar to the localized impurity states in doped semiconductors, localized modes will be created when defects are introduced into perfect photonic crystals (PCs).[1] Coupled cavity waveguides (CCWs) are formed by periodically placed defects, creating impurity bands through the coupling of defect modes.[2,3] They have exhibited potential applications in building high-efficiency waveguides and waveguide bends,[4] optical delay lines,[5−7] optical switches,[8,9] optical splitters, and waveguide intersections with low cross-talk,[10,11] etc.

In practice, disorders are inevitably introduced, more or less, in all kinds of fabrication processes for PCs. Accordingly, the effects of various disorders on the physical properties of PCs have been investigated, providing useful information for designing PC-based devices.[12−16] However, a detailed study of the influence of defect disorders on the properties of CCWs has not yet been conducted, although it is very important and useful for the design and improvement of CCW-based devices. Basically, the disorders in PCs can be classified into two types according to the localization theory. One is the disorder in defect size that belongs to Anderson disorder and the other is the disorder in period that can be considered as Lifshitz disorder. As for CCWs, the deviation in defect size is generally dominant because the defect period that is several times the lattice constant is easily controlled. Therefore, much attention has been paid to the control of defect size in the fabrication of CCWs because the deviation in defect size will lead to a marked reduction in the transmission of impurity bands.[17] From the viewpoint of designing CCWs, it is highly desirable to obtain impurity bands which are much less sensitive to the disorder of defects.

In this Letter, we generalize the guidance for designing CCWs with impurity bands having relatively large tolerance to defect disorder. For the sake of simplicity, we have chosen to study a one-dimensional (1D) CCW based on a 1D PC, as schematically shown in Fig. 1. The 1D PC is formed by a combination of GaAs ($n_1 = 3.4$) and air ($n_2 = 1.0$) layers with identical thickness of $0.50a$, where a is the lattice constant. The defects are introduced by periodically modifying the thickness of GaAs layers (for example, from $0.50a$ to $0.75a$). By increasing or decreasing the thickness of GaAs layers from their normal value, defect modes are created below the air band and above the dielectric band respectively. It has been indicated that the configuration of CCWs must fulfil certain conditions to obtain impurity bands with relatively flat transmission spectra.[7] If we denote the number of normal elements in between two neighbouring defects as n and the number of normal elements at the two ends of the waveguide as m, then a basic condition for achieving a quasiflat impurity band can be generalized as: (1) n is an even number and (2) $m = n/2$.

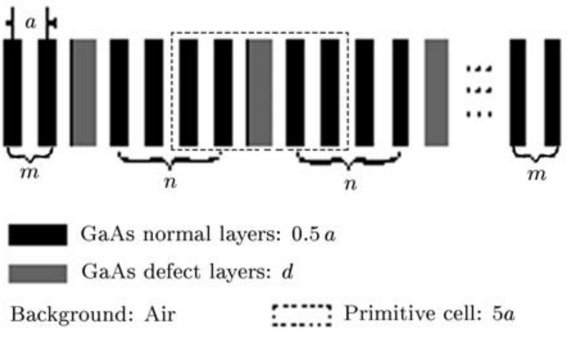

Fig. 1. Schematic of the 1D CCW studied in this paper. The PC atom forming the CCW is indicated by the dashed box.

First, let us inspect the primitive cell (a PC atom) forming the CCW as indicated in Fig. 1. Apparently,

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the resonant frequency of the PC atom is modified when the thickness of the defect layer is varied around its normal thickness. The defect thickness in a practically fabricated CCW generally follows a normal distribution characterized by a central thickness d_0 and a standard deviation σ. In frequency spectra, such a normal distribution of defect thickness results in a normal distribution of defect mode frequency that is generally referred to as the inhomogeneous broadening of defect modes.

We have calculated the defect mode frequency as a function of defect thickness, see Fig. 2. Obviously, the dependence of defect mode frequency on defect thickness varies greatly between different regimes. For $0 \le d \le 0.15a$, a small variation in defect thickness will result in a large change in mode frequency. In sharp contrast, defect modes are not sensitive to the change of defect thickness in the region of $0.25a \le d \le 0.40a$. Accordingly, a very weak dependence of mode frequency on defect thickness can be seen. As for increased-size defects ($d > 0.5a$), their dependence on defect size is moderate. Namely, the dependence of mode frequency on defect thickness (or in other words the sensitivity of mode frequency to defect size) determines the inhomogeneous broadening induced by the disorder in defect size. In Fig. 2, we compare the inhomogeneous broadenings induced by the same distribution of defect thickness for three types of defects, i.e. reduced-size defects whose frequencies located in the middle of the band gap ($d_0 = 0.10a$), reduced-size defects whose frequencies are close to the dielectric band ($d_0 = 0.30a$), and increased-size defects ($d_0 = 0.75a$). According to the above discussion, the largest inhomogeneous broadening is observed for $d_0 = 0.10a$ while the smallest one is found for $d_0 = 0.30a$.

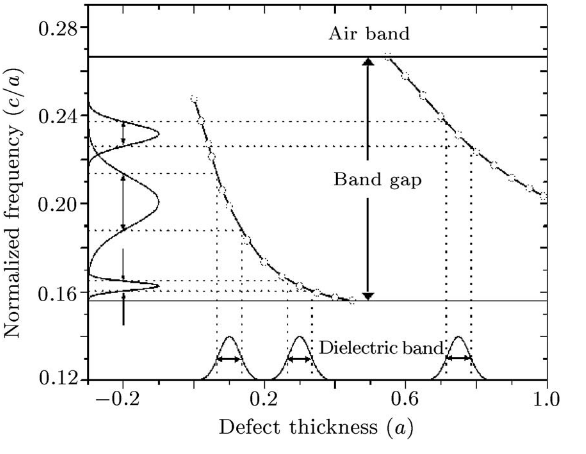

Fig. 2. Defect mode frequency as a function of defect thickness. A comparison of inhomogeneous broadening induced by the same distribution of defect thickness for three types of defects is also provided.

Before moving on to the investigation of the tolerance of impurity bands, we would like to discuss briefly the physical origin that governs the sensitivity of the defect modes. This is helpful for designing CCWs with impurity bands having relatively large tolerance to defect disorder. Again, we focus on the PC atoms shown in Fig. 1. If we denote the distribution of refractive index and the electric field corresponding to mode frequency ω as $n_0(z)$ and $E_\omega^{(0)}(z)$, then the eigenvalue equation reads as

$$H^{(0)}E_\omega^{(0)}(z) = (\omega/c)^2 E_\omega^{(0)}(z),$$

where $H^{(0)} = -\dfrac{1}{n_0^2(z)}\dfrac{\partial^2}{\partial z^2}$ is the Hamiltonian of the PC atom and c is the speed of light in vacuum. Here, we have set the origin of the coordinate at the centre of the defect layer. A small increase of defect thickness from d to $d+\delta d$ leads to a small change in the refractive index distribution. As a result, the Hamiltonian of the PC atom can be expressed as

$$H = H^{(0)} + H^{(1)},$$

where $H^{(1)}$ is a perturbation given by

$$H^{(1)} = \begin{cases} [(n_2/n_1)^2 - 1]H^{(0)}, \ z \in [-0.5(d+\delta d), -0.5d] \\ \qquad\qquad\qquad\quad \text{and } z \in [0.5d, 0.5(d+\delta d)] \\ 0 \qquad\qquad\quad \text{otherwise.} \end{cases}$$

The eigenvalue perturbation is obtained as follows:

$$\delta(\omega/c)^2 = \Big[\int E_\omega^{(0)*}(z)H^{(1)}E_\omega^{(0)}(z)dz\Big] \cdot \Big[\int E_\omega^{(0)*}(z)E_\omega^{(0)}(z)dz\Big]^{-1}.$$

Thus, the change of mode frequency $\delta\omega$ is found to be

$$\delta\omega = \omega[(n_2/n_1)^2 - 1] \cdot |E_\omega^{(0)}(z)|^2_{z=0.5d}\delta d/(2W),$$

where W is the total area energy density of the mode. It gives us some useful hints. One of them is that for the same δd, $|\delta\omega|$ is proportional to ω, implying that the defect modes locating in the lower part of the band gap are less sensitive to the variation of d for the 1D CCWs studied. It is in good agreement with what we have observed above.

On the other hand, it is well known that the formation of impurity bands originates from the coupling of the wavefunctions of localized defect modes. Apparently, the extension of wavefunction for a PC atom is best characterized by its quality factor Q representing the dissipation rate of the stored energy in the PC atom. The coupling is stronger for defects with lower quality factor due to the larger overlap of wavefunctions. We have calculated the Q factors of the defect modes (or PC atoms) as a function of defect size, as shown in Fig. 3. Also, we have provided the bandwidth of impurity bands formed by the corresponding defects. It can be seen that defect modes with lower Q factors will generate impurity bands with larger bandwidths. The defect modes located in the middle of the band gap possess larger Q factors while those close to the band edges have smaller Q factors. In addition, the Q factors for increased-size defects are much larger than those for reduced-size defects. Thus, it is easily understood why the coupling strength is stronger for the impurity bands formed by reduced-size defects, leading to a relatively wider bandwidth. The relatively wider bandwidth is partly responsible for the large tolerance of the impurity band near the dielectric band to the defect disorder as can be seen in the following.

Now we turn to the CCWs formed by different defects. By using the transfer matrix method, we first calculated the transmission spectra for the two types of CCWs containing 10 defects ($n = 4$, $m = 2$) in the absence of any disorder, as shown in Figs. 4(a) and 4(b) by the solid curves. At first glance, it seems that the quasiflat impurity band formed by the increased-size defect ($d_0 = 0.75a$) is desirable for the transmission of ultrashort pulses with broad frequency spectra. However, the impurity band is no longer a flat one as designed when the disorder in defects is considered. Here, only the disorder in defect size is considered because it is generally dominant in CCWs.[17] In accordance with practical fabrication, the normal distribution of the defect thickness can be expressed as $d = d_0 + \sigma\xi$, where ξ is a random variable which follows the standard normal distribution and σ is a standard deviation.

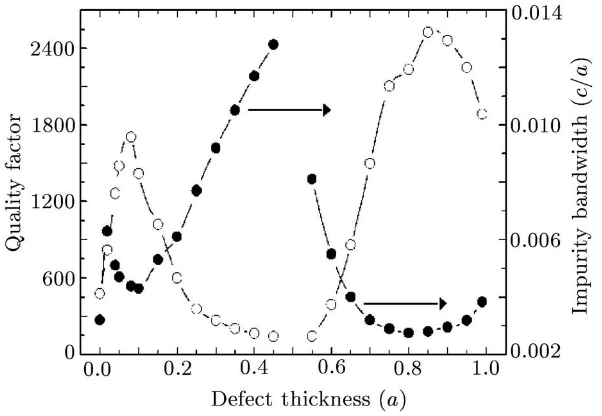

Fig. 3. Relationship between the quality factor of the defect mode and defect size in a PC atom. The bandwidths of the corresponding impurity bands formed by 10 defects are also provided.

As can be seen in Fig. 4(a), for the impurity band formed by the increased-size defect ($d_0 = 0.75a$), the quasiflat impurity band evolves into sharp resonant peaks and the transmittance at the valleys drops to -40 dB when a small deviation in defect size is introduced ($\sigma = 0.01$). As the deviation is further increased ($\sigma = 0.03$), only extremely sharp resonant modes with markedly reduced transmittance are left. In this case, we can say that the impurity band is completely destroyed. In sharp contrast, the impurity band formed by the reduced-size defect ($d_0 = 0.30a$) exhibits completely different behaviour when the defect disorder is introduced and increased, as can be seen in Fig. 4(b). For a small defect deviation ($\sigma = 0.01$), the spectral shape of the impurity band remains

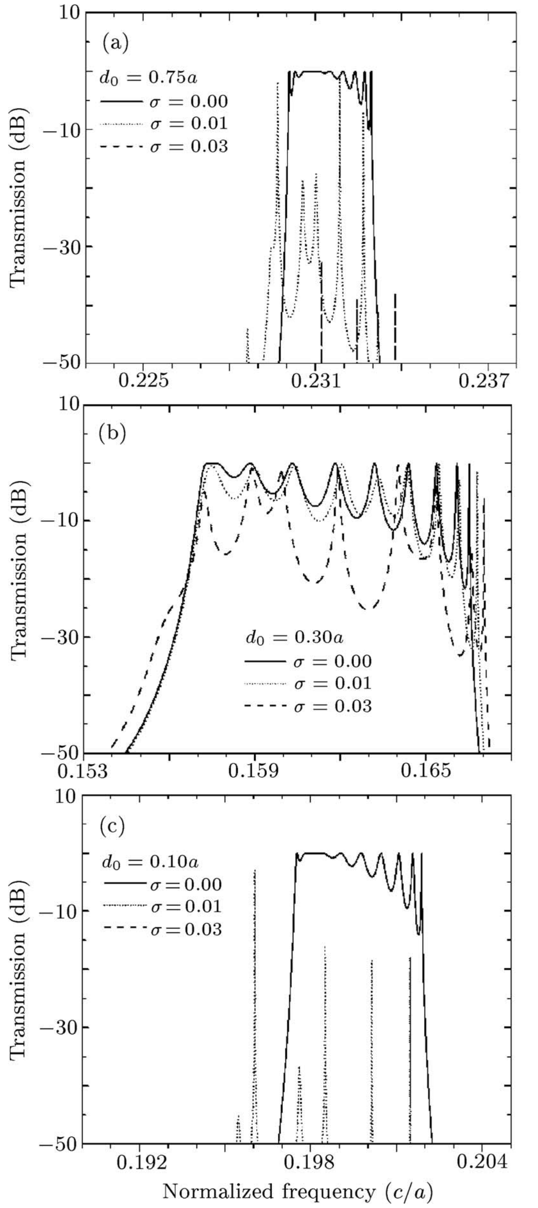

Fig. 4. Evolution of the transmission spectra of three CCWs containing 10 defects ($n = 4$, $m = 2$) upon the increase of defect disorder. The random variable used to generate defect thickness for the calculation of impurity bands is $\{-0.99, 1.28, -0.78, 0.89, -0.14, -0.31, -0.42, 0.41, 0.48, 1.29\}$. (a) An increased-size defect ($d_0 = 0.75a$), (b) a reduced-size defect whose frequency is close to the dielectric band ($d_0 = 0.30a$), (c) a reduced-size defect whose frequency located in the middle of the band gap ($d_0 = 0.10a$).

almost unchanged. Even for a large deviation ($\sigma = 0.03$), the impurity band still survives. These behaviours clearly indicate that the impurity band formed by the reduced-size defect close to the dielectric band is less sensitive to defect disorder than that formed by the increased-size defect.

From the viewpoint of defect coupling, the overlap of wavefunctions and thus the coupling strength is reduced when the resonant frequencies of two defect modes are selected to be different. According to the localization theory, the impurity band will be destroyed once the inhomogeneous broadening of the defect mode exceeds the bandwidth of the impurity band. In Fig. 2, it can be seen that the same deviation in defect size results in a larger inhomogeneous broadening for the increased-size defects than for the reduced-size defects close to the dielectric band. The interplay of narrower bandwidth and larger inhomogeneous broadening causes the impurity band to be easily deteriorated in the presence of defect disorder. Therefore, it is easily understood why the impurity band formed by the reduced-size defect near the dielectric band is so much less sensitive to the defect disorder.

It becomes clear that the tolerance of impurity bands is determined not only by the sensitivity of the resonant frequency to the change in defect size but also by the Q factors of the defects. The former governs the inhomogeneous broadening while the latter determines the bandwidth. Therefore, the tolerance of the impurity bands is finally determined by the competition of these two factors. As an example, let us inspect the defect mode of $d_0 = 0.10a$ whose Q factor is smaller than that of $d_0 = 0.75a$. The bandwidth of the corresponding impurity band is relatively wider, as shown in Fig. 4. However, its mode frequency is much more sensitive to the change of defect size as compared to the defect mode of $d_0 = 0.75a$, as can be seen in Fig. 2. As a consequence, the impurity band is destroyed much more rapidly upon the increase of defect disorder. For comparison, we have presented in Fig. 4(c) the evolution of the impurity band upon the increase of defect disorder. It can be seen that the impurity band evolves into extremely sharp resonant peaks even for a small deviation ($\sigma = 0.01$). For a large deviation ($\sigma = 0.03$), the impurity band vanishes and this implies that the maximum transmittance is below -50 dB. Therefore, it is the most vulnerable impurity band among the three impurity bands we have discussed above.

In conclusion, we have investigated the tolerance of impurity bands to the disorder of constitution defects in the 1D CCWs based on the transfer matrix method. It is found that the quasiflat impurity bands formed by increased-size defects are not the best choice in the presence of defect disorder. Instead, the impurity bands formed by reduced-size defects close to the dielectric band are found to be less-sensitive to defect disorder. It is revealed that the tolerance is determined not only by the Q factor of the constitutional defects but also by the sensitivity of the defect modes to the change of defect size.

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