



2024 年流体非线性偏微分方程研讨会 I

2024 Workshop on Nonlinear Partial Differential Equations in Fluids I

April 22, 2024 Mon (华师交叉中心 111 报告厅)		
Time	Lecturer and Title	Chair
09:30—10:30	Incompressible limit of ideal compressible Magneto-Hydrodynamics Paolo Secchi (University of Brescia)	Shijin Ding
10:30—11:30	Some results on free boundary problems of compressible fluids and related equations Tao Luo (City University of Hong Kong)	Paolo Secchi
12:00—14:30	Lunch	
14:30—15:30	Free interface problems for ideal incompressible MHD Sicheng Liu (University of Macau)	Alessandro Morando
15:30—17:30	Free discussion	
April 23, 2024 Tue (华师交叉中心 114 课室)		
09:00—10:00	On well-posedness of the two-dimensional MHD-Maxwell free interface problem Yury Trakhinin (Sobolev Institute of Mathematics)	Tao Luo
10:00—11:00	Nonlinear stability of two-dimensional compressible current-vortex sheets Paola Trebeschi (University of Brescia)	Yury Trakhinin
11:00—12:00	The two-dimensional plasma-vacuum interface problem in ideal MHD Alessandro Morando (University of Brescia)	Paola Trebeschi
12:00—14:30	Lunch	
14:30—17:30	Free discussion	



INCOMPRESSIBLE LIMIT OF IDEAL COMPRESSIBLE MAGNETO-HYDRODYNAMICS

PAOLO SECCHI

DICATAM, University of Brescia

paolo.secchi@unibs.it

Abstract. In this talk we consider the initial-boundary value problem in the halfspace for the system of equations of ideal Magneto-Hydrodynamics with a perfectly conducting wall boundary condition. We show the convergence of solutions to the solution of the equations of incompressible MHD as the Mach number goes to zero. Because of the characteristic boundary, where a loss of regularity in the normal direction to the boundary may occur, the convergence is shown in suitable anisotropic Sobolev spaces which take account of the singular behavior at the boundary.

About the lecturer. Paolo Secchi is Ordinary Professor at the University of Brescia (Italy) since 1996. He received a Laurea Degree in Mathematics in 1978 at University of Trento. Professor Secchi has long been engaged in the theory of nonlinear partial differential equations, especially from fluid mechanics. His research interests include nonlinear hyperbolic systems, mathematical theory of fluid dynamics, Navier-Stokes and Euler equations, and magnetohydrodynamics. He is the author of about 100 articles and given more than 100 talks in international conferences, universities and research centers.

SOME RESULTS ON FREE BOUNDARY PROBLEMS OF COMPRESSIBLE FLUIDS AND RELATED EQUATIONS

TAO LUO

City University of Hong Kong

Abstract. In this talk, I will first discuss the weak solutions of the free boundary problem for the full compressible Navier-Stokes with rough initial data for the motions with spherical symmetry in 3D in particular, and its behavior in small Mach number. This is based on joint work with Huihui Zeng. Then I may discuss some results for the free boundary problem of non-isentropic compressible Euler Equations coupled with a nonlinear Poisson equation, the emphasize will be on identifying stability conditions, based on joint work with Trivisa and Huihui Zeng.

About the lecturer. Prof. Tao Luo received his Ph.D. degree from the Institute of Mathematics, Chinese Academy of Sciences in 1995, and has been a professor in the Department of Mathematics, City University of Hong Kong since 2016. Prior to that,



he had been engaged in research and teaching at prestigious institutions such as the University of Michigan and Georgetown University, and was appointed as a professor at Georgetown University. Prof. Tao Luo's main research interests are in the analysis of nonlinear partial differential equations, including fluid free boundary problems, hyperbolic conservation laws, and variational principles. His recent research interests include free boundary problems of nonlinear partial differential equations in fluid dynamics and magnetic fluids. Prof. Luo Tao's research has been supported by the Rackham Research Grant of the University of Michigan, the CNR Research Grant of Italy, the CNRS Research Grant of France, the NSF Research Grant of the U.S.A., and the RGC Research Grant of Hong Kong, and the research results have been published in *Comm. Pure Appl. Math.*, *Arch. Rational Mech. Anal.*, *Comm. Math. Phys.*, *Adv. Math.* Prof. Tao Luo's major academic service positions include editorial board member of *Kinetic and Related Models* and editorial advisory board member of *Journal of Mathematical Physics*.

FREE INTERFACE PROBLEMS FOR IDEAL INCOMPRESSIBLE MHD

SICHENG LIU
University of Macau

Abstract. The motion of conducting fluids under the influence of magnetic fields is governed by MHD systems. In this talk, I will mainly discuss two types of free interface problems: one is the current-vortex sheet problems describing the motion of two plasmas separated by a free interface, the other is the plasma-vacuum interface problems depicting the motion of a plasma enclosed by a vacuum. I will focus on the local well-posedness theories in the standard Sobolev spaces without the graph assumption on the free interface. These works were completed under the guidance of Prof. XIN Zhouping at CUHK.

About the lecturer. Dr. Sicheng Liu received his Ph.D. degree from the Chinese University of Hong Kong in 2023. Dr. Sicheng Liu's main research interests are in the analysis of nonlinear partial differential equations, including fluid free boundary problems.



ON WELL-POSEDNESS OF THE TWO-DIMENSIONAL MHD-MAXWELL FREE INTERFACE PROBLEM

YURY TRAKHININ

Sobolev Institute of Mathematics

Abstract. We discuss the well-posedness of the 2D free interface problem in ideal compressible MHD taking into account the influence of the displacement current in vacuum, where the electric and magnetic fields satisfy the Maxwell equations. By using a suitable secondary symmetrization of the Maxwell system, we prove the well-posedness of the corresponding variable coefficient linearized problem, provided that at least one of the two unperturbed magnetic fields, in the plasma or in the vacuum region, is nonzero at each point of the interface.

About the lecturer. Professor Trakhinin is a Principal Researcher at the Sobolev Institute of Mathematics of the Russian Academy of Sciences (RAS) in Novosibirsk. He earned his Ph.D. in Mathematics from Novosibirsk State University in 1994, and he received his Doctor of Sciences degree (highest academic degree in Russia) in 2006. In 2018 he also received the honorary title of Professor of RAS, and in 2022 he was elected as a Corresponding member of RAS. In 1997-1998 he was an Alexander von Humboldt Research Fellow at the University of Stuttgart and in 2007 at the University of Leipzig (Germany). In 2002-2005 he worked as a Visiting Research Associate at the University of Hull (UK). In 2006, 2010 and 2012, he received a honorary fellowship of the Landau Network-Fondazione Alessandro Volta and Fondazione Cariplo and worked at the University of Brescia (Italy). His research interests include hyperbolic systems of

conservation laws, well-posedness theory for free boundary problems in mathematical fluid dynamics, shock waves and characteristic discontinuities. He is the author of about 80 papers and two monographs.

NONLINEAR STABILITY OF TWO-DIMENSIONAL COMPRESSIBLE CURRENT-VORTEX SHEETS

PAOLA TREBESCHI

DICATAM, University of Brescia

paola.trebeschi@unibs.it

Abstract. In this talk we are concerned with nonlinear stability and existence of two-dimensional current-vortex sheets in ideal compressible magnetohydrodynamics. This is a nonlinear hyperbolic initial-boundary value problem with characteristic free boundary. It is well-known that current-vortex sheets may be at most weakly (neutrally) stable due to the existence of surface waves solutions that yield a loss of derivatives in the energy estimate of the solution with respect to the source terms. We first identify a



sufficient condition ensuring the weak stability of the linearized current-vortex sheets problem. Under this stability condition for the background state, we show that the linearized problem obeys an energy estimate in anisotropic weighted Sobolev spaces with a loss of derivatives. Based on the weakly linear stability results, we then establish the local-in-time existence and nonlinear stability of current-vortex sheets by a suitable Nash-Moser iteration, provided the stability condition is satisfied at each point of the initial discontinuity. This result gives a new confirmation of the stabilizing effect of sufficiently strong magnetic fields on Kelvin-Helmholtz instabilities.

This is a joint work with A. Morando (Brescia), P. Secchi (Brescia) and D. Yuan (Beijing Normal Univ.)

References

- [1] A. Morando, P. Secchi, P. Trebeschi and D. Yuan, Nonlinear stability and existence of two-dimensional compressible current-vortex sheets, *Arch. Rational Mech. Anal.* (2023) 247:50, <https://doi.org/10.1007/s00205-023-01865-w>

About the lecturer. Paola Trebeschi is Associate Professor at the University of Brescia (Italy). She received a Laurea Degree in Mathematics in 1991 at University of Pavia (Italy) and a PhD in Mathematics in 1998 at University of Pisa (Italy). Professor Paola Trebeschi's research interests concern nonlinear system of hyperbolic equations with application to Fluid dynamics and Magnetohydrodynamics. In particular, she focuses on stability and local existence of piece-wise smooth weak solutions to compressible Euler system and compressible and incompressible Magnetohydrodynamics, which exhibit a free interface of strong discontinuities.

THE TWO-DIMENSIONAL PLASMA-VACUUM INTERFACE PROBLEM IN IDEAL MHD

ALESSANDRO MORANDO
DICATAM, University of Brescia

Abstract. In this talk we consider the two-dimensional plasma-vacuum interface problem in ideal compressible magnetohydrodynamics (MHD). This is a hyperbolic-elliptic coupled system with a characteristic free boundary. In the plasma region the 2D planar flow is governed by the hyperbolic equations of ideal compressible MHD, while in the vacuum region the magnetic field obeys the elliptic system of pre-Maxwell dynamics. At the free interface moving with the velocity of plasma particles, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary. The plasma-vacuum system is not isolated from the outside world, since it is driven by a given surface current which forces oscillations onto the system. We present our result



about the local-in-time existence and uniqueness of solutions to the nonlinear free boundary problem, provided that the plasma magnetic field or the vacuum magnetic field is non-zero at each point of the initial interface. The proof follows from the analysis of the linearized MHD equations in the plasma region and the elliptic system for the vacuum magnetic field, suitable tame estimates in Sobolev spaces for the full linearized problem, and a Nash-Moser iteration.

This is a joint work with P. Secchi (Brescia), Y. Trakhinin (Novosibirsk), P. Trebeschi (Brescia) and D. Yuan (Beijing Normal Univ.)

About the lecturer. Alessandro Morando is Associate Professor at University of Brescia (Italy). He is received a Laurea Degree in Mathematics in 1997 and a PhD in Mathematics in 2003. His research interests mainly address to nonlinear hyperbolic equations and systems, with applications to Fluid dynamics and Magneto-hydrodynamics. In particular, his current research focuses on the solvability of free boundary problems for compressible Euler equations and compressible or incompressible Magnetohydrodynamics.

