

We study the Dirichlet eigenvalue problem for the fractional sub-Laplacian $(-\Delta_{\mathbb{G}})^s$ on the homogeneous Carnot group $\mathbb{G} = (\mathbb{R}^n, \circ)$. Let Ω be a bounded open domain of \mathbb{G} and denote by λ_k ($:= \lambda_k(s)$) the k -th Dirichlet eigenvalue of the fractional sub-Laplacian operator $(-\Delta_{\mathbb{G}})^s$ on Ω . We use the abstract theory of Dirichlet forms and heat semigroups to construct explicit estimates for the trace of the Dirichlet heat kernel of $(-\Delta_{\mathbb{G}})^s$ via a comparison of heat kernels. Based on these estimates, we give an explicit lower bound estimate for λ_k , which exhibits the optimal growth order of k . Then, we establish the Weyl's law for the spectral counting function $N(\lambda)$. In particular, under a certain geometric condition of Ω , we also provide the remainder term estimate of the trace of the Dirichlet heat kernel and an explicit upper bound of λ_k with optimal growth order of k .