The Benefits of Third-party Logistics Firms as Financing Providers

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Abstract

We investigate the design of the supplier's wholesale price contract and the 3PL's joint logistics and financing services contract in a three-tier supply chain comprising a supplier, a 3PL firm, and a newsvendor-like retailer with capital constraints. The retailer can apply for bank financing or 3PL financing for purchasing when necessary. All members engage in a Stackelberg game with the supplier functioning as the leader. Our analysis indicates that the 3PL who finances the retailer with a low interest rate induces the retailer to order more, thereby causing the 3PL to obtain more profit from logistics services. The supplier benefits from 3PL financing by receiving a larger order from the retailer. Compared with bank financing, a retailer whose working capital level is not too low can benefit more with 3PL financing owing to lower purchasing (ordering and transportation costs) and financing costs. We further conclude that all members' optimal decisions remain unchanged when the 3PL is also capital constrained but can borrow from a bank. We examine the retailer and supplier's issues when the 3PL functions as the game leader instead of the supplier, and numerically demonstrate that the retailer and 3PL are better off while the supplier is worse off under 3PL leadership. Our results explain why 3PLs are willing to finance retailers' inventories in business practice and suggest that 3PLs should set low financing interest rates to improve channel performances.

Keywords: supply chain management, third-party logistics (3PL), newsvendor, capital constraints, financing service

1 Introduction

Shortage of funds is an important factor constraining the development and efficiency of supply chains. It is common for some supply chain participants to face capital constraints when making their operational decisions, especially small and medium-sized enterprises (SMEs) (Vandenberg 2003; Kouvelis and Zhao 2018). According to the American National Small Business Association's (NSBA) 2017 survey, 27% of small businesses cannot obtain adequate financing (McCracken and Barrera 2018). In Argentina, 23.1% of enterprises consider access to finance a major constraint, and in Greece, the percentage is as high as 35.9% (The World Bank 2019). Commercial bank loans are widely used by capital-constrained firms. However, strict credit histories, collateral requirements, and complicated application procedures are major barriers for SMEs in obtaining bank loans.

Financing services provided by business partners can effectively reduce information asymmetries and risk management difficulties related to borrowers, as lenders who interact directly with borrowers are more familiar with the borrowers, the value of products, and the markets than banks are. In recent years, third-party logistics providers (3PLs) have played an increasingly important role in supply chain operations as well as in alleviating SMEs' capital constraints. For example, UPS Capital, a UPS subsidiary, offers a variety of financial services such as package insurance, cargo insurance, cargo finance, and small business financing to UPS customers. Particularly, in international trade, if a firm is a customer of UPS Shipping Services and has capital constraints, by applying for UPS Capital cargo finance loans, it can obtain an advance rate of up to 100% of its supplier's commercial invoice and a payment term of up to 90 days without any collateral required (UPS Capital 2019). Eternal Asia is a large Chinese 3PL firm that serves supply chain enterprises. To support the development of small firms with budget constraints, Eternal Asia provides an integrated service to its upstream suppliers and downstream buyers, including distribution, sourcing, logistics, and financing. For downstream buyers, Eternal Asia orders from suppliers on behalf of them and delivers products to their warehouses. Buyers do not need to pay the suppliers directly since Eternal Asia pays during the ordering process, but repay Eternal Asia when they pick up the products from the warehouses (Yushang Financial 2020). Meanwhile, if necessary, buyers can also apply for short-term financing from Eternal Asia (Eternal Asia 2019), which can further alleviate their capital constraints. Similar services are also provided by other big 3PLs, such as Nippon Express (Nippon Express 2019) and Ingram Micro (Ingram Micro 2010).

With the advantages of information acquisition, industry experience, and access to low-cost capital, some large 3PLs have become supply chain organizers. Although 3PLs may benefit supply chains in many aspects, this study focuses on their important role in alleviating the supply chains' capital pressure. By investigating the ordering and transportation contracts' designs as well as different financing schemes, we focus on the efficiency improvements caused by 3PL financing and derive the conditions that benefit all supply chain members to ensure that this financing service is sustainable.

To achieve our goals, we consider a three-tier supply chain in which a supplier sells to a newsvendor-like retailer facing uncertain market demand, and a 3PL offers transportation and financing services to the retailer. All supply chain members are risk-neutral profit maximizers. The retailer has limited capital, which might be insufficient to cover the ordering and transportation fees, but can borrow from a bank or apply for 3PL financing services when necessary. In the 3PL financing service, the 3PL purchases from the supplier on behalf of the retailer and permits the retailer to pay a certain proportion of the purchasing and transportation fees at the end of the selling season with interest. We investigate the Stackelberg game between the supplier, 3PL, and retailer with regard to their decisions on ordering and transportation contracts. We assume that the supplier funtions as the leader, deciding the wholesale price first, with the 3PL functioning as the subleader who subsequently determines the transportation price and decides whether to finance the retailer. The 3PL also decides the interest rate if 3PL financing is provided. Given the preset contracts, the retailer simultaneously decides the order quantity and financing resource (bank or 3PL financing).

This study contributes to the literature in three ways. First, our work is among the first to comprehensively explore the application of 3PL financing schemes in a supply chain by incorporating both the wholesale price and transportation price as decision variables. Second, we demonstrate the effect of 3PL financing on the operations and profits of all the channel members and the entire supply chain. Compared with bank financing, the 3PL financing interest rate is lower and the retailer orders more, and hence the 3PL benefits from the retailer's large order and the efficiency of the supply chain is improved. This explains why 3PLs prefer financing retailers' inventories in business practice. For the supplier, it also benefits from the application of 3PL financing in the supply chain, which has not been addressed in previous studies. This study also finds that, compared with bank financing, under 3PL financing, the 3PL and retailer can withstand a higher wholesale price and purchasing cost, respectively. Third, in the extensions, we find that even if the 3PL is also capital-constrained and borrows from banks, our previous findings still hold. Additionally, when the 3PL functioning as the supply chain leader instead of the supplier, through comprehensive numerical experiments, we demonstrate that the supplier (or 3PL) has a first-mover advantage, which means it can obtain more profit when it functions as the leader rather than the subleader.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. We introduce the problems, assumptions, and model settings of this study in Section 3. In Sections 4 and 5, we study the respective models of the bank and 3PL financing the retailer. Section 6 analyzes the impact of 3PL financing on the supply chain and its members. In Section 7, we extend our model by considering the cases of the 3PL being capital-constrained and the 3PL functioning as the leader. Finally, we summarize the results and insights in Section 8.

2 Literature Review

In this study, we focus on capital-constrained supply chains where 3PLs play important roles. Hence, our work is closely related to two streams of literature: 3PL-provided transportation and financing services, and operational and financing strategies of supply chains.

The first stream of literature mainly focuses on exploring the important role of 3PLs in supply chains. Existing studies have shown that 3PLs can benefit supply chains in the aspects of information advantages (Wu 2004), relational advantages (Belavina and Girotra 2012), supply chain disruption risk management (Yang and Babich 2015), and integrated logistics and procurement services (Yang and Yu 2019). Please refer to Aguezzoul (2014) for a comprehensive literature review of the 3PLs selection problem. Chen et al. (2019b) address the impact of the 3PL's logistics service level and service price on the retailer's order quantity. In these studies, the researchers largely ignored the financing services offered by 3PLs. Chen et al. (2019a) investigate a payment scheme under which a

3PL obtains a payment delay arrangement from a supplier and then grants payment delay to a small retailer. Zhou et al. (2020) consider a scenario where a retailer applies for a bank loan and the loan is guaranteed by the upstream 3PL or supplier. They show that guarantor financing outperforms bank financing. Through case studies and interviews with some Chinese 3PLs, Li and Chen (2019) conclude that 3PLs can help firms obtain competitive advantages and improve their financial performances. Although the financial benefits brought by 3PLs are shown in these studies, the direct financing services provided by 3PLs are not considered. In this paper, we analyze the integrated logistics and financing services offered by 3PLs and theoretically prove and present the benefits that the 3PL financing scheme provides to supply chain members.

Operational management and corporate financing interfaces in supply chains have recently received substantial interest (Giannetti et al. 2011; Jing et al. 2012; Kouvelis and Zhao 2018; Chen et al. 2019a). Many studies have investigated capital-constrained suppliers and the mitigation of suppliers' financial distress through financing schemes such as bank financing and buyer financing (Tunca and Zhu 2018; Deng et al. 2018; Tang et al. 2018). Nevertheless, we are particularly interested in studies in which commercial bank financing and trade credit financing are the financing schemes commonly adopted by capital-constrained retailers. For example, Buzacott and Zhang (2004), Dada and Hu (2008), and Alan and Gaur (2018) investigate the financing strategies of retailers when they can financing from profit-maximizing banks. When bank loans are competitively priced for relevant risks, Kouvelis and Zhao (2011) examine the impact of different bankruptcy costs on a retailer's ordering and financing strategies; Kouvelis and Zhao (2016) study the channel coordination problem under some commonly adopted contracts, such as revenue-sharing, buyback, and quantity discount. When multiple financing channels are available, Kouvelis and Zhao (2012) and Jing et al. (2012) analyze the supplier's and retailer's decisions under both bank financing and trade credit, and compare their decisions under the two financing channels. Kouvelis and Zhao (2012) find that trade credit stimulates the retailer to order more compared with bank financing, but Jing et al. (2012) find that trade credit is less attractive than bank financing for the retailer because the supplier charges a wholesale price when offering trade credit. Furtherly, Yang and Birge (2018) point out that trade credit allows the retailer to share partial demand risk with the supplier and hence improves supply chain efficiency.

The aforementioned studies have shown that bank financing or trade credit can effectively alleviate the negative effects of funds shortage in supply chains. And from the perspective of methodology, our work is closely related to Kouvelis and Zhao (2012). While, this paper contributes to this stream of study by considering a three-tier supply chain with a different financing scheme, i.e., 3PL financing. 3PL financing can be very helpful for SMEs to alleviate their capital pressure, especially when they are unable to obtain bank loans or trade credit. Against this background, we explore the design of the optimal 3PL financing scheme and disclose its advantages in alleviating the retailer's capital pressure and improving supply chain efficiency.

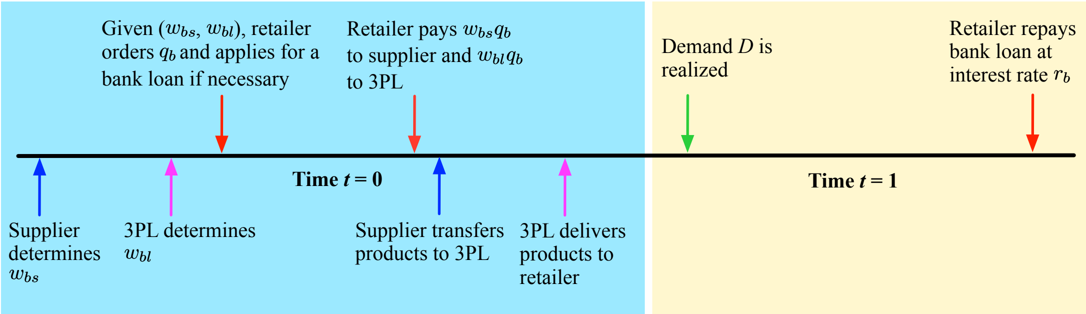
The mostly related literature to our work are Chen and Cai (2011) and Huang et al. (2019). Chen and Cai (2011) seminally study 3PL financing of a retailer in a three-tier supply chain. They consider the 3PL as a credit providers and investigate its decision on the interest rate. Their analysis shows that compared with bank financing, 3PL financing brings higher profits to all channel members and the entire supply chain. However, they ignore the supplier's wholesale price decision as well as the 3PL's pricing decision on logistics services. Huang et al. (2019) extend Chen and Cai (2011) by incorporating the supplier's wholesale price decision and providing the condition the wholesale price and 3PL financing interest rate should satisfied that leading to supply chain coordination. In our work, we extend these two studies by offering a comprehensive discussion about the 3PL's decisions on pricing logistics and financing services as well as the supplier's decision on wholesale price. When the decisions on wholesale and transportation prices are considered, different from Chen and Cai (2011), we find that 3PL financing does not always dominate bank financing for the 3PL and retailer, but in contrast find that 3PL financing is influenced by the value of wholesale and transportation prices. Additionally, we also point out that even 3PL financing can improve the supply chain performance, the three-tier supply chain coordination cannot be achieved, which is different from Huang et al. (2019).

3 Problem Description

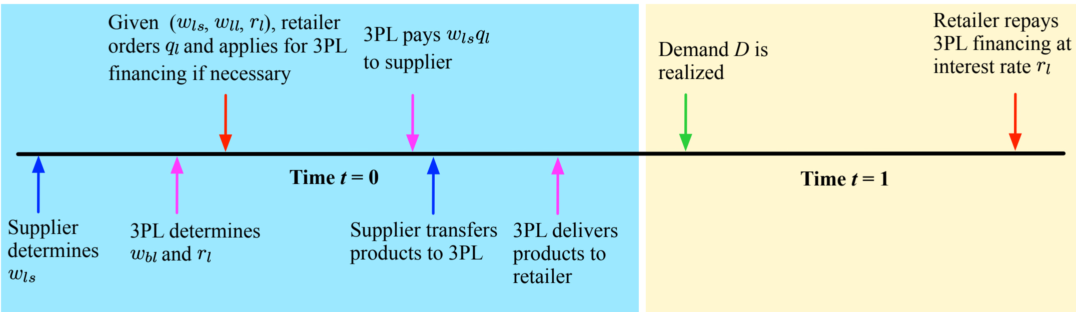
In this study, we consider a three-tier supply chain comprising a supplier, a third-party logistics provider (3PL), and a retailer with limited working capital. The retailer faces the newsvendor problem and needs to satisfy the uncertain market demand in one selling season consisting of two stages indexed by $t = 0$ and $t = 1$. The three supply chain members engage in a Stackelberg game, with the supplier functioning as the leader.

3.1 Sequence of Events

The sequence of events is as follows. At time $t = 0$, the supplier first offers a wholesale price contract to the retailer where the wholesale price is w_s. Then, the 3PL determines the unit transportation price w_l and the interest rate of 3PL financing r_l simultaneously. Consequently, given w_s, w_l, and r_l, the retailer determines order quantity q and pays the supplier fully, that is, $w_s q$. Upon receiving the order, the supplier manufactures the products, which the 3PL then transports to the retailer. The retailer pays the 3PL a transportation fee $w_l q$ for the logistics service. For the convenience of expression, we refer to $(w_s q + w_l q)$ as the retailer's unit purchasing cost. The retailer we consider in this study is a small or medium-sized enterprise with limited working capital y. When it does not have enough capital to fully cover its purchasing cost, it can choose to apply for a bank loan or 3PL financing service (see Figure 1(a) for the bank financing scheme and 1(b) for the 3PL financing scheme). If the retailer chooses a short-term bank loan, it receives money $(w_s + w_l)q - y$ instantly at a fixed interest rate r_b and pays the full ordering and transportation costs. If the retailer chooses 3PL financing, it gives all its working capital y to the 3PL, and the 3PL pays the retailer's remaining costs by charging an interest rate r_l. After reserving enough money for operations at $t = 0$, all the members invest their leftover funds, if any, at the risk-free interest rate $r_f \geq 0$.



(a) Bank financing



(b) 3PL financing

Figure 1: Two financing schemes

At time $t = 1$, the market demand is realized and the retailer sells the products to the customers at retail price p. We assume that unmet demand is lost without any penalty and the salvage value of the unsold products at the end of $t = 1$ is 0. Additionally, if the retailer applies for a bank loan or 3PL financing at $t = 0$, the retailer is obliged to repay the loan or financing with interest. Irrespective of the financing scheme chosen, we assume that the retailer is a limited liability enterprise and it repays the loan with the cash on hand (i.e., the sales revenue) and collateral assets S at the end of $t = 1$. If the retailer cannot repay the loan fully, it has to announce bankruptcy after transferring all its remaining assets to the bank or 3PL.

3.2 Notations and Assumptions

To distinguish the notations under the two financing schemes, we use subscript $i = b, l$ to represent the bank financing scheme and 3PL financing scheme respectively. The supplier's decision variable is the wholesale price w_{is}, and the 3PL's decision variables are the transportation price w_{il} and interest rate r_l. Let q_i be the retailer's order quantity and r_b be the bank's interest rate. We also constrain the interest rates r_b and r_l to be greater than or equal to the risk-free interest rate r_f. All the interest rates we mentioned in this paper are defined over the period from time 0 to 1.

The unit production and transportation costs are denoted by c_s and c_l respectively. We assume $(w_{is}+w_{il})(1+r_f) \leq p$ and $(w_{is}+w_{il})(1+r_l) \leq p$ to ensure that the retailer has an incentive to order from the supplier and apply for 3PL financing when necessary. Additionally, to ensure that both the supplier and 3PL will obtain non-negative profits in the wholesale and transportation processes, we assume $c_s \leq w_{is}$ and $c_l \leq w_{il}$.

The market demand, represented by D, is unknown to all supply chain members at $t = 0$. However, its probability density function (p.d.f.) $f(\cdot)$ and cumulative distribution function (c.d.f.) $F(\cdot)$ are common knowledge. Let the failure rate $h(\cdot) = f(\cdot)/\bar{F}(\cdot)$, where $\bar{F}(\cdot) = 1-F(\cdot)$. In this study, we focus on demand distributions with increasing and concave failure rates. This characteristic captures some commonly used distributions, such as truncated normal, uniform, exponential, power, and Weibull $f(D) = k\lambda^k D^{k-1}e^{(\lambda D)^k}$ ($\lambda > 0$, $k \geq 2$) (Zhou 2009). Other assumptions are summarized as follows. (A1) All the supply chain members are risk-neutral. (A2) Each supply chain member's objective is to maximize its expected profit during the selling season. (A3) The financial market is perfect (no taxes, transaction costs, or bankruptcy costs) and the competition is perfect. (A4) The bank, supplier, and 3PL have enough capital and face no bankruptcy risks, but the retailer may face bankruptcy risks if it applies for financing from the bank or 3PL.

Referring to assumption A1, as the owner of a SME with capital constraints, the retailer may have different risk preferences in practice. The findings about the risk preference of entrepreneurs or the owners of small firms are mixed. March and Shapira (1987) and Vereshchagina and Hopenhayn (2009) show that entrepreneurs are risk-seeking. Caliendo and Kritikos (2009) and Koudstaal et al. (2016) show that they are risk-averse, but have a lower degree of risk aversion than managers and employees. In addition, Hu (2014) reports that entrepreneurs are more likely to be risk-neutral. In this paper, in order to capture the main characteristics of the 3PL financing and purchasing contracts while keeping the model tractable, we assume the cash-constrained retailer is risk-neutral. Referring to assumption A2, some empirical studies have shown that some regions, such as Canada, Switzerland, Spain, South Africa, Chile, Ireland, Saudi Arabia, etc., have (nearly-) perfectly competitive bank markets (Shaffer 1993; Al-Muharrami et al. 2006; Bikker and Spierdijk 2009). We present the notation used throughout this paper in Table 1.

4 Model under Bank Financing

We consider the bank financing model as our benchmark model. In this section, we explore the supply chain members' and the bank's decisions when the retailer applies for a bank loan when it needs more money, i.e., $i = b$. First, we determine the bank's interest rate, and then examine the other players' problems by backward induction.

Table 1: Summary of Notation

D	Market demand with p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$
$[0, N]$	Support of D
$h(\cdot)$	Failure rate of the market demand distribution, $h(D) := f(D)/\overline{F}(D)$
p	Retail price of the product
y	Retailer's working capital level
S	Value of retailer's collateral assets at the end of the selling season
i	Index of bank financing scheme ($i = b$) and 3PL financing scheme ($i = l$)
q_i	Retailer's order quantity
w_{is}	Wholesale price determined by the supplier
w_{il}	Unit transportation price determined by the 3PL
c_s	Supplier's unit production cost
c_l	3PL's unit transportation cost
r_f	Risk-free interest rate
r_i	Bank loan interest rate ($i = b$) or 3PL financing interest rate ($i = l$)
B_i	Amount of money the retailer borrowed
z_i	Minimum market demand for the retailer to fully repay the loan
π_i	Retailer's expected profit
Π_i	Supplier's expected profit
Γ_i	3PL's expected profit

4.1 Bank Loan Interest Rate

At time $t = 0$, the retailer's order quantity is q_b and the money it needs to cover all relevant costs is $(w_{bs} + w_{bl})q_b$. Since the retailer has limited working capital, we define $B_b := [(w_{bs} + w_{bl})q_b - y]^+$ as the amount of money needed from the bank. If y is large enough to cover all costs, $B_b = 0$, otherwise, $B_b > 0$. At the end of $t = 1$, the retailer has assets $L_b := p\min(q_b, D) + [y - (w_{bs} + w_{bl})q_b]^+(1 + r_f) + S$, where $\min(q_b, D)$ represents the sales volume. Then the money that the retailer can repay the bank equals

$$\min(L_b, B_b(1 + r_b)). \tag{1}$$

Based on the assumption that the financial market is perfect and the bank loan is competitively priced, the bank's optimal interest rate r_b satisfies

$$\mathbb{E}[\min(L_b, B_b(1 + r_b))] = B_b(1 + r_f). \tag{2}$$

We refer to Chen and Wan (2011) Proposition 1 for the proof of the existence and uniqueness of r_b, satisfying Equation (2). It is worth noting that the value of r_b is unknown until the bank receives the retailer's financing request and knows the retailer's order quantity. Equation (2) just offers the bank a principle to determine its interest rate, and the value of r_b is affected by many factors, such as q_b, y, and S. When the retailer needs financing (i.e., $(w_{bs} + w_{bl})q_b > y$) and $y + S/(1 + r_f) < (w_{bs} + w_{bl})q_b$ is satisfied, $r_b > r_f$; otherwise, $r_b = r_f$.

4.2 Retailer's Order Decision under Bank Financing

Given the wholesale price w_{bs} and transportation price w_{bl}, the retailer needs to determine the order quantity. Under bank financing, when the retailer's order quantity is q_b, its expected profit is

$$\pi_b(q_b) = \mathbb{E}\left[L_b - \min(L_b, B_b(1+r_b))\right] - y(1+r_f) - S. \tag{3}$$

$L_b - \min(L_b, B_b(1+r_b))$ is the retailer's total assets at the end of $t = 1$. Meanwhile, since the retailer invested its working capital and collateral assets in the business at $t = 0$, considering the time value of money, $y(1+r_f) + S$ is the retailer's total cost. By substituting the expression of L_b and Equation (2) in Equation (3), we can calculate the retailer's optimal order quantity in this newsvendor model, which is

$$q_b = \bar{F}^{-1}\left(\frac{(w_{bs} + w_{bl})(1+r_f)}{p}\right). \tag{4}$$

Unlike the result of the classic newsvendor model, $(w_{bs} + w_{bl})(1 + r_f)$ stands for the retailer's unit purchasing cost because the time value of money is considered.

4.3 3PL's Pricing Decision under Bank Financing

Under the bank financing scheme, given the wholesale price w_{bs} and considering the retailer's optimal response function, the 3PL only needs to determine the transportation price w_{bl}. At $t = 0$, after bearing the transportation cost $c_l q_b$ and receiving the transportation revenue $w_{bl} q_b$ from the retailer, the 3PL invests its leftover money with risk-free interest rate r_f. Then the 3PL's profit is

$$\Gamma_b(w_{bl}) = (w_{bl} - c_l) q_b (1 + r_f). \tag{5}$$

Equation (4) indicates that one-to-one mapping between w_{bl} and q_b exists. Then we have

$$w_{bl} = p\bar{F}(q_b)/(1+r_f) - w_{bs}. \tag{6}$$

By substituting the expression of w_{bl} in Equation (5), the 3PL's objective function can be rewritten as a function of q_b, which is $\Gamma_b(q_b) = p\bar{F}(q_b)q_b - (w_{bs} + c_l)q_b(1 + r_f)$. Then, we can derive the optimal q_b from the 3PL's perspective, and obtain the optimal w_{bl} by substituting the optimal q_b in Equation (6). By analyzing the first- and second-order derivatives of $\Gamma_b(q_b)$ with respect to q_b, the optimal q_b satisfies

$$p\bar{F}(q_b)[1 - q_b h(q_b)] - (w_{bs} + c_l)(1 + r_f) = 0, \tag{7}$$

and $q_b \in [0, q_\alpha)$, where q_α solves $q_\alpha h(q_\alpha) = 1$.

4.4 Supplier's Pricing Decision under Bank Financing

Upon receiving the retailer's order and payment, the supplier starts manufacturing at unit cost c_s and invests its leftover money with risk-free interest rate. Then, the supplier's profit is

$$\Pi_b(w_{bs}) = (w_{bs} - c_s)q_b(1 + r_f). \tag{8}$$

From Equation (7), we express w_{bs} as $w_{bs}(q_b) = p\bar{F}(q_b)[1-q_b h(q_b)]/(1+r_f)-c_l$ for $q_b \in [0, q_\alpha)$. Taking the first-order derivative of $w_{bs}(q_b)$ with respect to q_b, we have $dw_{bs}/(q_b)dq_b < 0$. Thus, a one-to-one mapping between w_{bs} and q_b exists. By substituting $w_{bs}(q_b)$ in Equation (8), we can rewrite the supplier's profit as a function of the order quantity, that is,

$$\begin{aligned} \Pi_b(q_b) = p\bar{F}(q_b)q_b[1 - q_b h(q_b)] - (c_s + c_l)q_b(1 + r_f), \\ \text{s.t.} \quad q_b \in [0, q_\alpha). \end{aligned} \tag{9}$$

By analyzing the derivatives of $\Pi_b(q_b)$ with respect to q_b and combined with our previous analysis, we summarize all the players' optimal decisions in the following proposition.

Proposition 1 *If the retailer chooses bank financing, the equilibrium order quantity q_b^*, transportation price w_{bl}^*, wholesale price w_{bs}^*, and bank loan interest rate r_b^* satisfy the following equation system:*

$$\begin{aligned} &p\bar{F}(q_b^*)\left\{[1 - q_b^* h(q_b^*)]^2 - q_b^*\left[h(q_b^*) + q_b^* h'(q_b^*)\right]\right\} - (c_s + c_l)(1 + r_f) = 0, \\ &w_{bl}^* = \frac{p\bar{F}(q_b^*)}{1 + r_f} - w_{bs}^*, \\ &w_{bs}^* = \frac{p\bar{F}(q_b^*)[1 - q_b^* h(q_b^*)]}{1 + r_f} - c_l, \\ &\mathbb{E}\left[\min\left(L_b, B_b(1 + r_b^*)\right)\right] = B_b(1 + r_f). \end{aligned}$$

We can observe that all supply chain members' optimal decisions are independent of r_b^*, y, and S. That is, the financing problem does not influence supply chain operations, and hence, the financing and operational problems can be analyzed separately. This conclusion is consistent with the result of the wholesale price contract design in a two-tier capital-constrained supply chain discussed by Kouvelis and Zhao (2012).

5 Model under 3PL Financing

In this section, we explore the financing and operational problems in the three-tier supply chain when the retailer chooses to apply for 3PL financing, i.e., $i = l$. The supplier who functions as the game leader decides the wholesale price w_{ls}, the 3PL not only determines the transportation price w_{ll}, but also the financing interest rate r_l. Given w_{ls}, w_{ll}, and r_l, the retailer determines the order quantity

q_l. Solving the game by backward induction, we first consider the retailer's problem.

5.1 Retailer's Decision under 3PL Financing

When the retailer's order quantity is q_l, the amount of money the 3PL should pay for the retailer's purchasing cost is defined as $B_l := [(w_{ls} + w_{ll})q_l - y]^+$. If the initial capital level y is high enough to cover all relevant costs, $B_l = 0$ and the retailer will invest its leftover money with risk-free interest rate r_f; otherwise, the retailer needs to apply for 3PL financing and $B_l > 0$. At the end of the selling season, the retailer obtains sales revenue $p \min(q_l, D)$ and needs to repay the 3PL at interest rate r_l. Then the retailer's expected profit equals

$$\pi_l(q_l) = \mathbb{E}\left\{p \min(q_l, D) + [y - (w_{ls} + w_{ll})q_l]^+(1 + r_f) + S - B_l(1 + r_l)\right\}^+ - y(1 + r_f) - S. \tag{10}$$

The retailer faces bankruptcy risks if the market demand level is very low and it cannot fully repay the debt. Then, what is the lowest market demand such that the retailer can repay the debt fully? We define z_l as the lowest demand such that the retailer does not need to declare bankruptcy, which equals

$$z_l := \frac{\{[(w_{ls} + w_{ll})q_l - y]^+(1 + r_l) - S\}^+}{p}. \tag{11}$$

We call z_l the retailer's bankruptcy threshold. When the retailer applies for 3PL financing, if $D < z_l$, the retailer will go bankrupt because the demand is too low; if $D \geq z_l$, the demand is high enough for the retailer to pay back the 3PL in full and will not go bankrupt. By comparing z_l with q_l, we obtain Lemma 1.

Lemma 1 *Under 3PL financing,* $z_l < q_l$.

Lemma 1 implies that the retailer's bankruptcy threshold is always lower than its order quantity. If $z_l \geq q_l$, the retailer will always go bankrupt and lose assets $y(1 + r_f) + S$. Consequently, it will not apply for 3PL financing. The relationship $z_l < q_l$ ensures that the retailer has an incentive to apply for 3PL financing when necessary.

According to different relationships between y, S, and $(w_{ls}+w_{ll})q_l$, Equation (10) can be rewritten as

$$\pi_l(q_l) = \begin{cases} \mathbb{E}[p \min(q_l, D) + S - B_l(1 + r_l)]^+ - y(1 + r_f) - S, & \text{if } (w_{ls} + w_{ll})q_l > y, \quad (12a) \\ \mathbb{E}\left[p \min\left(q_l, D\right)\right] - y(1 + r_f), & \text{if } (w_{ls} + w_{ll})q_l = y, \quad (12b) \\ \mathbb{E}[p \min(q_l, D)] - (w_{ls} + w_{ll})q_l(1 + r_f), & \text{if } (w_{ls} + w_{ll})q_l < y. \quad (12c) \end{cases}$$

Equation (12a) represents the case when the retailer needs to apply for 3PL financing. In this case, at the end of the selling season, the retailer repays the debt with all its assets ($p \min(q_l, D) + S$). Note that if the retailer cannot fully repay the 3PL, the retailer has to transfer all its assets to the 3PL and declare bankruptcy. Equation (12b) represents the case when the retailer has just enough working capital for purchasing to maintain normal operations. Equation (12c) represents the case when the retailer has more than adequate capital and has money left after paying related costs with its initial working capital.

Given w_{ls}, w_{ll}, and r_l, by analyzing the derivatives of Equation (12), we obtain the retailer's optimal order quantities under different cases, which are summarized as follows.

Proposition 2 *Given w_{ls}, w_{ll}, and r_l, the retailer's optimal order quantity is*

$$q_l = \begin{cases} \bar{F}^{-1}\left(\frac{(w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)}{p}\right), & \text{if } (w_{ls}+w_{ll})q_l > y, & (13a) \\ \frac{y}{w_{ls}+w_{ll}}, & \text{if } (w_{ls}+w_{ll})q_l = y, & (13b) \\ \bar{F}^{-1}\left(\frac{(w_{ls}+w_{ll})(1+r_f)}{p}\right), & \text{if } (w_{ls}+w_{ll})q_l < y. & (13c) \end{cases}$$

In the case of bank financing, we show that the retailer's order quantity is independent of its assets and the bank loan interest rate because bank loans are competitively priced. However, with 3PL financing, according to Equation (13a), the order quantity is affected by y, S, and r_l. The underlying reason for this is that the 3PL, unlike the bank, determines r_l with the goal of maximizing its expected profit.

Now, we know the retailer's optimal response functions under different scenarios in Proposition 2. Furthermore, to simplify the analysis of the 3PL and supplier's problems in this section's following part, we need to transfer the constraints on $(w_{ls}+w_{ll})$ and q_l in Equation (13) to q_l only.

Lemma 2 *The three constraints in Equations (13a)-(13c) are equivalent to the constraints $q_l \in \Omega_j$, j =1, 2, 3, respectively, where Ω_j are the q_l sets satisfying Inequalities (14a)-(14c), respectively.*

$$\begin{cases} \bar{F}(q_l)q_l > \frac{y(1+r_l)}{p}, & (14a) \\ \frac{y(1+r_f)}{p} \leq \bar{F}(q_l)q_l \leq \frac{y(1+r_l)}{p}, & (14b) \\ \bar{F}(q_l)q_l < \frac{y(1+r_f)}{p}. & (14c) \end{cases}$$

Note that $\bar{F}(q_l)q_l$ is a quasi-concave function and its maximum value is achieved at q_α, which solves $q_\alpha h(q_\alpha) = 1$. Then, combined with Figure 2, Ω_j, j =1, 2, 3, can be expressed as $\Omega_1 = (q_1^l, q_1^u)$, $\Omega_2 = [q_2^l, q_1^l] \cup [q_1^u, q_2^u]$, and $\Omega_3 = [0, q_2^l) \cup (q_2^u, N]$. Here, q_1^l and q_1^u are the solutions of $\bar{F}(q_l)q_l = y(1+r_l)/p$, q_2^l and q_2^u are the solutions of $\bar{F}(q_l)q_l = y(1+r_f)/p$. The inequalities $0 \leq q_2^l \leq q_1^l \leq q_\alpha \leq q_1^u \leq q_2^u \leq N$ always hold. Ω_1 indicates the order quantity interval where the retailer has capital constraints and needs to apply for 3PL financing. In Ω_2, the retailer does not apply for financing but has no money left after paying the purchasing cost. In Ω_3, the retailer has sufficient working capital to cover related costs. Next, we analyze the effects of w_{ls}, w_{ll}, and r_l on the retailer's order quantity and expected profit.

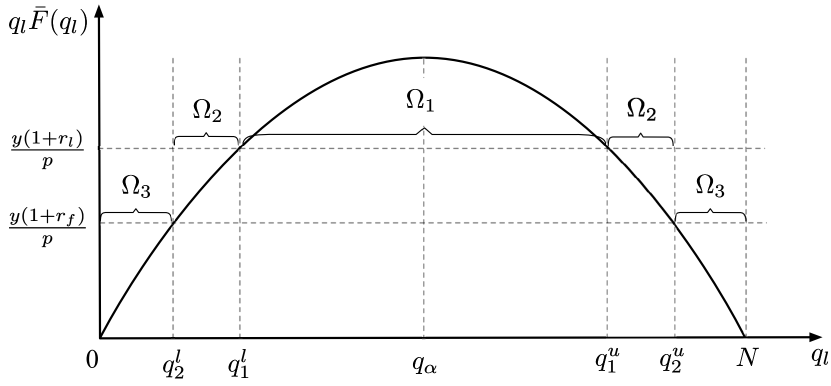


Figure 2: Values of q_l under different retailer capital levels.

Lemma 3 *Under 3PL financing,*

(i) given r_l, π_l and q_l monotonously decrease in $(w_{ls}+w_{ll})$;

(ii) given r_l and w_{ls}, π_l and q_l monotonously decrease in w_{ll};

(iii) given w_{ls} and w_{ll}, when $(w_{ls}+w_{ll})q_l > y$, π_l and q_l monotonously decrease in r_l.

Lemma 3 indicates that a lower wholesale price, transportation price, or financing interest rate will encourage the retailer to order more products, which will lead to greater profit. Lemma 3(i) indicates that, given r_l, one-to-one mapping between $(w_{ls}+w_{ll})$ and q_l exists; Lemma 3(ii) indicates that when both r_l and w_{ls} are fixed, one-to-one mapping between w_{ll} and q_l exists. We will use these two mappings to explore the 3PL and supplier's optimal decisions in the following analysis.

5.2 3PL's Decision under 3PL Financing

When the retailer applies for 3PL financing, the 3PL needs to decide both the transportation price w_{ll} and interest rate r_l. In this study, we assume that the 3PL is a large supply chain service company, such as UPS or Eternal Asia, capable of providing both logistics and financing services to supply chains.

Given the retailer's limited working capital at time $t=0$, the amount of money the 3PL receives from the retailer is $\min(y,(w_{ls}+w_{ll})q_l)$. Subsequently, the 3PL spends $(w_{ls}+c_l)q_l$ on providing the retailer with ordering and logistics services. If $(w_{ls}+c_l)q_l \le y < (w_{ls}+w_{ll})q_l$, the retailer needs to apply for financing but y is enough for the 3PL to cover the ordering and transportation costs $(w_{ls}+c_l)q_l$. However, if $y<(w_{ls}+c_l)q_l$, the 3PL has to spend its own money $[(w_{ls}+c_l)q_l - y]$ for related costs. Finally, at the end of the selling season, the 3PL receives $\min\{p\min(q,D)+S,((w_{ls}+w_{ll})q_l-y)^+(1+r_l)\}$ from the retailer. Then, we can calculate the 3PL's expected profit, which is

$$\begin{aligned}\Gamma_l(w_{ll}, r_l) =& \mathbb{E}\{[\min(y, (w_{ls}+w_{ll})q_l) - \min(y, (w_{ls}+c_l)q_l)](1+r_f)\} \\ &- \mathbb{E}\{[(w_{ls}+c_l)q_l - y]^+(1+r_f)\} \\ &+ \mathbb{E}\left\{\min\left\{p\min(q_l, D) + S, [(w_{ls}+w_{ll})q_l - y]^+(1+r_l)\right\}\right\}.\end{aligned} \tag{15}$$

The first line of Equation (15) indicates the amount of money that the 3PL can invest with risk-free interest rate at time $t = 0$; the second line is the extra money (except y) that the 3PL spends during the ordering and transportation processes; the third line indicates the money that the 3PL can receive from the retailer at the end of $t = 1$.

Expanding Equation (15) gives the objective function of the 3PL under the different working capital levels of the retailer, as shown in Equation (16). Equations (16a)-(16c) represent the 3PL's expected profit when the retailer needs financing, when financing is unnecessary but all the working capital is used for purchasing, and when the retailer has sufficient capital, respectively.

$$\Gamma_l(w_{ll}, r_l) = \begin{cases} \mathbb{E}\left\{\min\left\{p\min(q_l, D) + S, [(w_{ls}+w_{ll})q_l - y](1+r_l)\right\}\right\} \\ \quad - [(w_{ls}+c_l)q_l - y](1+r_f), & \text{if } q_l \in \Omega_1, \quad (16a) \\ [y - (w_{ls}+c_l)q_l](1+r_f), & \text{if } q_l \in \Omega_2, \quad (16b) \\ (w_{ll}-c_l)q_l(1+r_f), & \text{if } q_l \in \Omega_3. \quad (16c) \end{cases}$$

In Lemma 3(ii), we show that one-to-one mapping between q_l and w_{ll} exists. Then, by substituting the inverse function of Equation (13) in (16), we obtain the 3PL's decision function regarding q_l and r_l, i.e., $\Gamma_l(q_l, r_l)$. Next, we keep r_l fixed and discuss the optimal decision of the 3PL regarding q_l for a given r_l. Once the optimal q_l is obtained, we can calculate the optimal transportation price w_{ll} through the one-to-one mapping. Please note that order quantity is not the 3PL's or supplier's decision variable, but the order quantity is affected by their pricing decisions. By analyzing the first- and second-order partial derivatives of $\Gamma_l(q_l, r_l)$ with respect to q_l and defining $\delta_l := (w_{ls}+w_{ll})(1+r_l)qh(z_l)/p$ for $z_l > 0$, we derive Lemma 4.

Lemma 4 *Given w_{ls}, r_l and the increasing concave function $h(\cdot)$, the optimal order quantity q_l for the 3PL is $\hat{q}$ or $\overline{q}$, where $\hat{q}$, $\bar{q} \in [0, q_a)$, $\hat{q}$ satisfies*

$$\frac{p\bar{F}(\hat{q})[1 - \hat{q}h(\hat{q})]}{1-\delta_l} - (w_{ls}+c_l)(1+r_f) = 0, \tag{17}$$

and $\overline{q}$ satisfies

$$p\bar{F}(\bar{q})[1 - \bar{q}h(\bar{q})] - (w_{ls}+c_l)(1+r_f) = 0. \tag{18}$$

According to Lemma 4, the 3PL limits the retailer's order quantity in the interval $[0, q_a)$. For the

3PL, if the optimal $q_l = \hat{q}$, the optimal transportation price w_{ll} can be obtained by substituting $\hat{q}$ in Equation (13a); if the optimal $q_l = \bar{q}$, the optimal w_{ll} can be derived by substituting $\bar{q}$ in Equation (13c). However, the q_l in Lemma 4 is obtained based on a given r_l. Next, we take r_l into consideration and explore the 3PL's optimal transportation price and interest rate simultaneously. For this purpose, we define $\underline{\mathbb{C}} = \left[p\bar{q}\bar{F}(\bar{q}) - S\right]/(1+r_f)$ and $\overline{\mathbb{C}} = \left[p\hat{q}\bar{F}(\hat{q}) - S\right]/(1+r_f)$ as a lower level and a higher level of the retailer's working capital respectively, and $\underline{\mathbb{C}} < \overline{\mathbb{C}}$.

Proposition 3 *Given w_{ls} and the increasing concave function $h(\cdot)$, under 3PL financing,*

(i) the 3PL's optimal financing interest rate is $r_l^* = r_f$;

(ii) for the 3PL, if $y < \underline{\mathbb{C}}$, the optimal $q_l = \hat{q}$; if $y > \overline{\mathbb{C}}$, the optimal $q_l = \bar{q}$; if $\underline{\mathbb{C}} \le y \le \overline{\mathbb{C}}$, the optimal $q_l = \arg\max\left(\Gamma_l(\bar{q}, r_f), \Gamma_l(\hat{q}, r_f)\right)$; the optimal transportation price can be obtained by substituting q_l in Equation (13).

Proposition 3(i) indicates that the 3PL will set the interest rate as low as the risk-free interest rate. In particular, when $y < (w_{ls} + w_{ll})q_l - S/(1+r_f)$, $r_l^* < r_b$. That is, the 3PL has an incentive to relieve the retailer's capital pressure and the financing service offered by the 3PL is cheaper than bank loans. The 3PL undertakes a higher risk than the bank because it sets the financing rate as the risk-free rate. As we will show in the following analysis, by doing this, the 3PL can reduce the retailer's financing pressure and induce it to order more products. Thus, the 3PL can obtain additional profits by providing transportation services, which cover the costs caused by the retailer's default risks. Therefore, the 3PL has incentives to set a low interest rate when financing the retailer.

In Proposition 3(ii), the values of $\hat{q}$ and $\bar{q}$ can be obtained from Equations (17) and (18) respectively by setting $r_l = r_f$. When $y < \underline{\mathbb{C}}$, the retailer is cash-strapped, so it needs to apply for financing and has bankruptcy risks; when $y > \overline{\mathbb{C}}$, the retailer may or may not require financing, but there is no bankruptcy risk even financing is needed. For $y > \overline{\mathbb{C}}$, it is worth noting that if the wholesale prices under the bank and 3PL financing schemes are the same, i.e., $w_{ls} = w_{bs}$, combined with Equation (7), we can conclude that $\bar{q} = q_b^*$ holds and both the 3PL's and the retailer's decisions are the same as those when the retailer is financed by the bank. Moreover, when $\underline{\mathbb{C}} \le y \le \overline{\mathbb{C}}$, the retailer's working capital level is relatively low. The decision on whether to apply for financing and whether bankruptcy risks exist are sensitive to the wholesale and transportation prices. In this situation, the 3PL needs to compare the two solutions $\hat{q}$ and $\bar{q}$, and choose the one with higher expected profit, which is, $q_l = \arg\max\left(\Gamma_l(\bar{q}, r_f), \Gamma_l(\hat{q}, r_f)\right)$.

5.3 Supplier's Decision under 3PL Financing

As the leader of the supply chain, the supplier needs to determine the product's wholesale price w_{ls}. Since the 3PL provides financing service to the retailer, the supplier can always receive full payment for the order. Then the supplier manufactures the product at unit cost c_s and invests its remaining funds at the risk-free rate r_f. As a result, the supplier's profit is

$$\Pi_l\left(w_{ls}\right) = \left(w_{ls} - c_s\right) q_l \left(1 + r_f\right). \tag{19}$$

To analyze the supplier's problem, the 3PL's and retailer's response functions for a given wholesale price w_{ls} should be considered. However, when the retailer has different levels of working capital, the 3PL's and retailer's response functions differ. Hence, we need to discuss the supplier's optimal decision corresponding to the retailer's various working capital levels.

Lemma 5 *Combining Equations (17) - (19), we have*

(i) q_l *monotonously decreases in* w_{ls}*;*

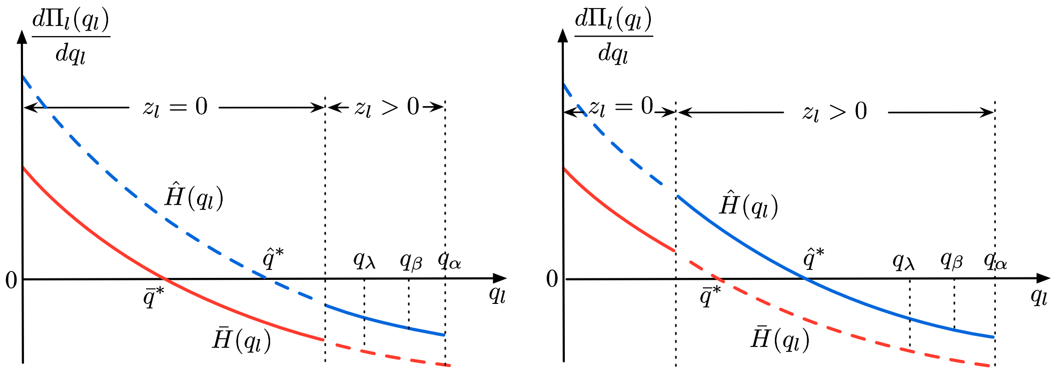
(ii) as shown in Figure 3, $\hat{\mathrm{H}}(q_l) > \bar{\mathrm{H}}\left(q_l\right) \geq 0$ *for* $q_l \in [0, \bar{q}^*]$*;* $\bar{\mathrm{H}}\left(q_l = q_\lambda\right) < 0$ *and* $\hat{\mathrm{H}}\left(q_l = q_\beta\right) < 0$*; and* $d\bar{\mathrm{H}}\left(q_l\right)/dq_l < 0$ *for* $q_l \in [0, q_\alpha)$*.*

In Lemma 5, $\hat{\mathrm{H}}\left(q_l\right) := d\Pi_l\left(q_l\right)/dq_l$ when $y < (w_{ls}+w_{ll})q_l - S/(1+r_f)$, $\bar{\mathrm{H}}\left(q_l\right) := d\Pi_l\left(q_l\right)/dq_l$ when $y \geq (w_{ls} + w_{ll})q_l - S/(1 + r_f)$. $\hat{q}^*$ and $\bar{q}^*$ are the solutions of $\hat{H}(\hat{q}^*) = 0$ and $\bar{H}(\bar{q}^*) = 0$, respectively. When $w_{ls} = c_s$, q_β is the solution of Equation (17) and q_λ is the solution of Equation (18), and we have $q_\lambda < q_\beta < q_\alpha$.

Lemma 5(i) indicates that one-to-one mapping between q_l and w_{ls} exists regardless of whether the retailer requires financing. Therefore, we can express w_{ls} as a function of q_l according to Equations (17) and (18), and then substitute it in Equation (19) to analyze the optimal order quantity q_l from the supplier's perspective. By calculating the optimal q_l for the supplier and substituting it in Equation (17) or (18), the supplier's optimal wholesale price can be obtained.

In Lemma 5(ii), since $\hat{\mathrm{H}}(q_l) > \bar{\mathrm{H}}(q_l) \geq 0$ for $q_l \in [0, \bar{q}^*]$, it is obvious that $\hat{q}^* > \bar{q}^*$. As can be seen in Figure 3, the optimal order quantity for the supplier may be $\bar{q}^*$ or $\hat{q}^*$, and we know that $\bar{q}^* < q_\lambda$, $\hat{q}^* < q_\beta$. In our previous analysis in Subsection 5.2, we show that the 3PL constrains the order quantity in the interval of $[0, q_\alpha)$. However, based on the inequalities $q_\lambda < q_\beta < q_\alpha$, we find that the supplier will further reduce the order quantity of the supply chain due to the increase of the supply chain's echelon. It is worth noting that $\bar{H}(q_l)$ monotonically decreases in q_l. Thus, when $\bar{q}^*$ is a feasible solution, it is also unique. However, as it is unable to figure out the monotonicity of $\hat{H}(q_l)$, $\hat{q}^*$ may not be unique when it is feasible. When there are multiple $\hat{q}^*$, let $\hat{q}^* = \arg\max\{\Pi_l\left(q_l\right)|\hat{\mathrm{H}}\left(q_l\right) = 0\}$, and $\hat{q}^*$ is the supplier's optimal order quantity when the retailer has bankruptcy risks.

In Subsection 5.2 we defined $\underline{\mathbb{C}} = \left[p\bar{q}\bar{F}(\bar{q}) - S\right]/\left(1 + r_f\right)$ and $\overline{\mathbb{C}} = \left[p\hat{q}\bar{F}(\hat{q}) - S\right]/\left(1 + r_f\right)$, where both $\bar{q}$ and $\hat{q}$ are functions of w_{ls}. As the leader of the supply chain, by determining the value of w_{ls}, the supplier indirectly controls the equilibrium values of $\bar{q}$ and $\hat{q}$, i.e., $\bar{q}^*$ and $\hat{q}^*$. Therefore, we redefine $\underline{\mathbb{C}}$ and $\overline{\mathbb{C}}$ as $\underline{\mathbb{C}} := \left[p\bar{q}^*\bar{F}(\bar{q}^*) - S\right]/\left(1 + r_f\right)$ and $\overline{\mathbb{C}} := \left[p\hat{q}^*\bar{F}(\hat{q}^*) - S\right]/\left(1 + r_f\right)$, respectively. The supplier's optimal decision is summarized in Proposition 4.



(a) When $\bar{q}^*$ is feasible (b) When $\bar{q}^*$ is infeasible

Figure 3: Changes of $d\Pi_l\left(q_l\right)/dq_l$ in q_l

Proposition 4 *Given the increasing concave function* $h(\cdot)$, *when* $y < \underline{\mathbb{C}}$, $q_l^* = \hat{q}^*$, *when* $y > \overline{\mathbb{C}}$, $q_l^* = \bar{q}^*$, *and when* $\underline{\mathbb{C}} \le y \le \overline{\mathbb{C}}$, $q_l^* = \arg\max\left(\Pi_l\left(\bar{q}^*\right), \Pi_l\left(\hat{q}^*\right)\right)$. *The optimal wholesale price can be derived by substituting* q_l^* *in Equation (17) or (18).*

The supplier will set wholesale prices based on the retailer's various working capital levels. When $y < \underline{\mathbb{C}}$, since q_l decreases in w_{ls} and $q_l^* = \hat{q}^* > \bar{q}^*$, we find that the supplier sets a lower wholesale price to stimulate the retailer into ordering more, and the order quantity in this case is even larger than that when the retailer has sufficient working capital.

Combined with Propositions 2-4, the equilibrium order quantity, financing interest rate, transportation price, and wholesale price under the 3PL financing scheme are summarized as follows.

(1) If $\Pi_l\left(\hat{q}^*\right) > \Pi_l\left(\bar{q}^*\right)$, $q_l^* = \hat{q}^*$, $r_l^* = r_f$, $w_{ll}^* = \frac{p\bar{F}(\hat{q}^*)}{\bar{F}(z_l)(1+r_f)} - w_{ls}^*$, and $w_{ls}^* = \frac{p\bar{F}(\hat{q}^*)(1-\hat{q}h(\hat{q}^*))}{(1-\delta_l)(1+r_f)} - c_l$, $\hat{q}^*$ satisfies

$$\frac{p\bar{F}(\hat{q}^*)[1-\hat{q}^*h(\hat{q}^*)]^2}{1-\delta_l} + \left[\frac{\bar{F}(\hat{q}^*)[1-\hat{q}^*h(\hat{q}^*)]^2h(z_l)}{\bar{F}(z_l)(1-\delta_l)^2} - h(\hat{q}^*)\right]\frac{p\hat{q}^*\bar{F}(\hat{q}^*)}{1-\delta_l}$$
$$+\left[\frac{\bar{F}(\hat{q}^*)^2[1-\hat{q}^*h(\hat{q}^*)]^2h'(z_l)}{\bar{F}(z_l)^2(1-\delta_l)^2} - h'(\hat{q}^*)\right]\frac{p\hat{q}^{*2}\bar{F}(\hat{q}^*)}{1-\delta_l} = (c_s+c_l)(1+r_f).$$

(2) Otherwise, $q_l^* = \bar{q}^*$, $r_l^* = r_f$ and $(q_l^*, w_{ll}^*, w_{ls}^*)$ is the same as $(q_b^*, w_{bl}^*, w_{bs}^*)$ in Proposition 1.

6 Bank Financing versus 3PL Financing

In Sections 4 and 5, we explore all the channel members' optimal operational decisions under bank financing and 3PL financing respectively. In this section, we compare the two financing schemes from the perspectives of the entire supply chain and different decision makers, and analyze their preferences for the two financing schemes.

6.1 Supply Chain has Higher Efficiency under 3PL Financing

We first explore the benefit that the 3PL financing service provides to the entire supply chain by comparing the order quantities under 3PL and bank financing schemes, and examine whether supply chain coordination, i.e., the maximal channel profit, can be achieved. We denote q_c as the order quantity when the supply chain is coordinated, where q_c satisfies

$$\bar{F}(q_c) = \frac{(c_s + c_l)(1 + r_f)}{p}. \tag{20}$$

Proposition 5 *Compared with bank financing, the supply chain obtains a larger order quantity under 3PL financing, i.e., $q_l^* \geq q_b^*$, but cannot achieve coordination, i.e., $q_b^* \leq q_l^* < q_c$.*

From the classic newsvendor problem, we know the entire supply chain's profit is a concave function of the order quantity, and the highest profit is realized at q_c. When the order quantity is less than q_c, the supply chain's profit increases in the order quantity. From Proposition 5, we know that channel coordination cannot be achieved, i.e., $q_i^* < q_c$, with a wholesale price contract under either of the financing schemes due to the double marginalization effect and decentralized decision-making in the supply chain. However, the supply chain can obtain a larger order when the financing scheme is switched from bank to 3PL financing, thereby increasing the supply chain's overall profit. The 3PL financing scheme can be considered as a mechanism that partially transfers the market demand risk faced by the retailer to the 3PL in the form of the 3PL's bad debt risk by offering the retailer a low financing interest rate. Consequently, the retailer is willing to order more products and the supply chain efficiency improves.

6.2 Consistent Selections between the Two Financing Schemes

Next, we compare the two financing schemes from the perspective of each member in the supply chain. As the leader of the supply chain, the supplier first determines the product's wholesale price. The supplier's pricing decision directly reflects its preference between the two financing schemes. Given the supplier's decision, the 3PL determines the transportation price and financing interest rate. The 3PL's decisions similarly reflect its preference between the two financing schemes. Finally, the retailer determines the order quantity and selects a financing scheme. Regarding the selection of financing schemes, the entire supply chain can achieve a stable equilibrium only when all the members' preferences are consistent. Otherwise, the upstream and downstream members have to readjust their decisions until a consensus is reached.

When the retailer has sufficient working capital or needs financing but has no bankruptcy risk, the problems under the two financing schemes are the same. Hence, we will only consider the situation in which the retailer needs financing and has bankruptcy risks. Through Proposition 6, we show that all members' selections between the two financing schemes are consistent.

Proposition 6 *When the retailer needs financing and has bankruptcy risks,*

(i) for the supplier, the 3PL financing scheme strictly dominates the bank financing scheme, i.e., $\Pi_l(w_{ls}^*) > \Pi_b(w_{bs}^*)$;

(ii) for the 3PL, given the supplier's decision w_{ls}^*, the 3PL financing scheme strictly dominates the bank financing scheme, i.e., $\Gamma_l(w_{ll}^*, r_l^* | w_{ls}^*) > \Gamma_b(w_{bl}^* | w_{ls}^*)$*;*

(iii) for the retailer, given $(w_{ls}^*, w_{ll}^*, r_l^*)$*, the 3PL financing scheme strictly dominates the bank financing scheme, i.e.,* $\pi_l(q_l^* | w_{ls}^*, w_{ll}^*, r_l^*) > \pi_b(q_b^* | w_{ls}^*, w_{ll}^*, r_l^*)$.

From Proposition 6(i), we know that the supplier always prefers 3PL financing. However, if the supplier sets the wholesale price as w_{ls}^* but the 3PL or retailer prefers bank financing, the supplier will not obtain the expected profit under 3PL financing. Fortunately, under this supplier (leader)-3PL (sub-leader)-retailer (follower) game sequence, the second and third parts of Proposition 6 indicate that, given the supplier's offer w_{ls}^*, the 3PL and retailer will consequently select 3PL financing, which guarantees that the supplier can obtain its optimal profit.

Under the Stackelberg game setting, the supplier and 3PL can adjust their pricing decisions as they want and hence, the selections of the financing schemes among the channel members are consistent. The results in Proposition 6 can be applied to monopoly markets or monopolistic competition markets where a firm provides differentiated products/services and has absolute or some degree of market power. In the following Subsection 6.3, we will further reveal that the selection is still consistent even if the supplier cannot adjust the wholesale price.

6.3 Conditions under which 3PL Financing is Sustainable

Although all the members' choices between the two financing schemes are the same, the 3PL and retailer's preferences are influenced by the supplier's wholesale price decision. Without the influence of the supplier's leader power, their preferences might differ. In other words, the 3PL and retailer might be negatively impacted when the switch is made from bank to 3PL financing, and hence they may prefer a supplier offering wholesale price w_{bs}^* but not w_{ls}^*. Therefore, it is important to figure out whether 3PL financing can lead to Pareto improvements for all the members so that 3PL financing is sustainable.

Next, we explore the conditions under which the 3PL and retailer can obtain greater expected profits under 3PL financing rather than bank financing. Define Θ, which is a small non-negative term and solves

$$\mathbb{E}\left\{p\min\left[\frac{(w_{bs}^* + \Theta + w_\varphi)\, q_b^* - y}{p}(1+r_b^*), D\right]\right\} = \mathbb{E}\left[p\min(z_b, D)\right] + \Theta q_b^*(1+r_f),$$

where w_φ satisfies

$$(w_{bs}^* + \Theta + w_\varphi)\,(1+r_b^*)\bar{F}\left(\frac{(w_{bs}^* + \Theta + w_\varphi)q_b^* - y}{p}(1+r_b^*)\right) = (w_{bs}^* + w_{bl}^*)(1+r_f).$$

Define Δ, which is a small non-negative term and solves $\mathbb{E}\left[p\min(q_b^*, D)\right] - \left[(w_{bs}^* + w_{bl}^*)\, q_b^* - y\right](1+r_f) =$

$\mathbb{E}\left\{p\min(q_\phi, D) - [(w_{bs}^* + w_{bl}^* + \Delta) q_\phi - y](1+r_f)\right\}^+$, where q_ϕ satisfies $p\bar{F}(q_\phi) = (w_{bs}^* + w_{bl}^* + \Delta)(1+r_f)\bar{F}\left(\frac{[(w_{bs}^*+w_{bl}^*+\Delta)q_\phi - y](1+r_f)-S}{p}\right)$. Then we have the following proposition.

Proposition 7 *For the 3PL, if $w_{ls}^* < w_{bs}^* + \Theta$, $\Gamma_l(w_{ll}^*, r_l^*) > \Gamma_b(w_{bl}^*)$. For the retailer, if $w_{ls}^* + w_{ll}^* < w_{bs}^* + w_{bl}^* + \Delta$, $\pi_l(q_l^*) > \pi_b(q_b^*)$.*

From Proposition 7, we observe that the 3PL will obtain greater expected profit as long as the wholesale price under 3PL financing is not too high compared with bank financing. The underlying reason for this is that as the retailer purchases more products under 3PL financing, the 3PL can obtain greater profit by providing logistics services. If w_{ls}^* is much higher than w_{bs}^*, the 3PL has little room to adjust the transportation price, and the retailer may decrease the order quantity due to the high ordering costs. As a result, the 3PL will be negatively impacted by 3PL financing. Similarly, the retailer can obtain greater expected profit under 3PL financing as long as the unit purchasing cost (wholesale price plus transportation price) is not too high because it can reduce financing costs by adopting 3PL financing and obtain more profit by increasing the order quantity.

Proposition 7 also implies that compared with bank financing, the 3PL and retailer can withstand a higher wholesale price and purchasing cost under 3PL financing, which may benefit them practically. In particular, in a situation where the wholesale and transportation prices are affected by factors such as raw material prices or labor costs but unaffected by the choice of financing schemes, even if the two prices increase slightly due to such factors, the retailer and 3PL can still obtain greater profit by switching from bank to 3PL financing.

Recall that Proposition 6 is obtained based on the Stackelberg game setting in which both the supplier and 3PL can adjust their pricing decisions according to their preferred financing schemes. Combining with Proposition 7, we observe that even when the supplier cannot adjust the wholesale price, i.e., $w_{ls} = w_{bs}$, 3PL financing is preferred by the 3PL and retailer.

Proposition 8 *There exists a capital level threshold $\underline{\underline{\mathbb{C}}}$, where $0 < \underline{\underline{\mathbb{C}}} < \underline{\mathbb{C}}$, such that if $\underline{\underline{\mathbb{C}}} \le y \le \bar{\mathbb{C}}$, $w_{ls}^* + w_{ll}^* \le w_{bs}^* + w_{bl}^*$ and $\pi_l(q_l^*) \ge \pi_b(q_b^*)$. Additionally, if $y \to 0$, $w_{ls}^* + w_{ll}^* \to p/(1+r_l^*)$ and $\pi_l(q_l^*) \to 0$.*

Proposition 8 indicates that for a retailer who is not very constrained in cash, i.e., $\underline{\underline{\mathbb{C}}} \le y \le \bar{\mathbb{C}}$, obtains no less expected profit under 3PL financing than under bank financing due to lower unit purchase and financing costs. However, when the retailer is cash strapped with nearly zero working capital, the purchasing cost will be very high and it will obtain nearly zero profit. In this case, the retailer is worse off under 3PL financing.

Combining Propositions 7 and 8, we find that the 3PL and retailer will obtain less profit in certain cases when the financing scheme is switched from bank to 3PL financing. They may prefer a supplier offering the wholesale price w_{bs}^*, and hence the retailer will apply for bank financing. However, for

Table 2: All members' optimal decisions and expected profits under bank financing and 3PL financing for $y \in [100, 1000]$.

y	w^*_{is}	w^*_{il}	$w^*_{is} + w^*_{il}$	q^*_i	r^*_i	Π_i	Γ_i	π_i	Financing
				Bank loan ($i = b$)					
100	62.62	21.31	83.93	11.87	6.14%	282.03	141.00	70.50	
200	62.62	21.31	83.93	11.87	5.74%	282.03	141.00	70.50	
300	62.62	21.31	83.93	11.87	5.39%	282.03	141.00	70.50	
400	62.62	21.31	83.93	11.87	5.14%	282.03	141.00	70.50	
500	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	Yes
600	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	
700	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	
800	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	
900	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	
1000	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	No
				3PL loan ($i = l$)					
100	63.94	23.12	87.06	18.59	5.00%	467.31	196.25	46.70	
200	63.95	21.96	85.91	18.21	5.00%	457.95	185.19	56.12	
300	63.99	20.82	***84.81***	17.85	5.00%	449.59	172.69	***66.19***	
400	64.04	19.71	***83.75***	17.51	5.00%	442.04	159.36	***77.07***	
500	63.98	18.74	82.72	17.29	5.00%	435.17	147.27	89.24	Yes
600	63.98	17.75	81.73	17.03	5.00%	428.88	133.01	102.08	
700	62.89	17.64	80.53	17.56	5.00%	422.06	137.66	120.22	
800	60.23	18.74	78.96	19.12	5.00%	406.20	172.41	146.94	
900	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	
1000	62.62	21.31	83.93	11.87	5.00%	282.03	141.00	70.50	No

the supplier, setting the wholesale price as w^*_{bs} is not the best strategy because 3PL financing strictly dominates bank financing (Proposition 6(i)).

Next, we conduct a numerical study to compare the two financing schemes for different working capital levels of the retailer from all supply chain members' perspectives. Assume that the market demand follows a uniform distribution $U(0, 100)$. Set retail price $p = 100$, the supplier's unit production cost $c_s = 40$, the 3PL's unit transportation cost $c_l = 10$, the risk-free interest rate $r_f = 5\%$, the retailer's working capital level $y \in [100, 1000]$ and collateral assets $S = 500$. The numerical results are presented in Table 2.

Under bank financing, given Proposition 1, all the members' profits and related decisions are independent of the retailer's working capital y. Hence, the wholesale price, transportation price, and order quantity under bank financing ($i = b$) are not changed as y increases from 100 to 1000. However, the bank loan interest rate is affected by the retailer's working capital level. When $y \in [100, 400]$, the retailer needs financing and has bankruptcy risks. Hence, the bank sets $r^*_b > r_f$. When $y \in [500, 900]$, the retailer needs financing but $r^*_b = r_f$ because the retailer has no bankruptcy risk. When $y = 1000$, the retailer has enough working capital and does not need financing. Under 3PL financing ($i = l$), when the retailer's working capital $y \leq 900$, it has capital constraints and requires financing from the 3PL, but the bankruptcy risk $z^*_l > 0$ for $y \in [100, 800]$ and $z^*_l = 0$ for $y = 900$. When $y = 1000$,

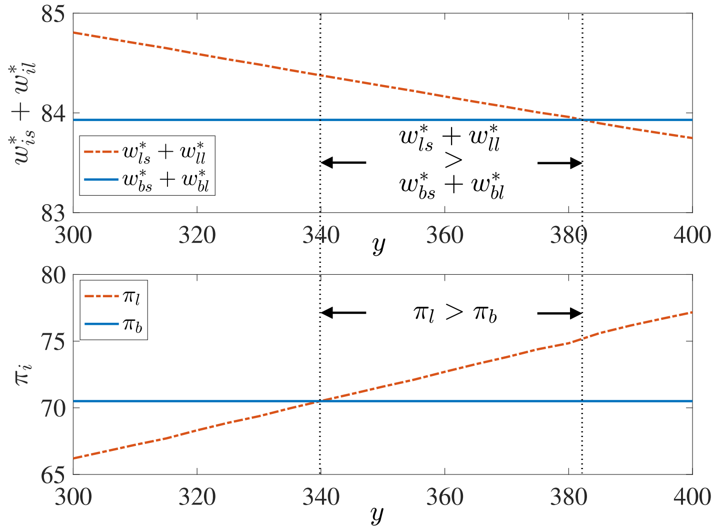


Figure 4: Changes of $w^*_{is} + w^*_{il}$ and π_i, $i = l, b$, for $y \in [300, 400]$.

the retailer has sufficient working capital to cover the purchasing cost. From the columns q^*_i and Π_i, we can observe that the order quantity and supplier's expected profit are higher under 3PL financing compared with bank financing, which are consistent with the results in Proposition 5 and 6(i).

In Proposition 7, we show that the 3PL benefits from 3PL financing if the wholesale price is not too high. The w^*_{is} column shows that even though the optimal wholesale price w^*_{ls} is higher than w^*_{bs} when $y \leq 500$, the 3PL still obtains a higher profit under 3PL financing. For the retailer with $y \leq 800$, under the case of 3PL financing, we observe that its expected profit increases as it has more working capital. When $400 \leq y \leq 800$, the results show that $w^*_{ls} + w^*_{ll} < w^*_{bs} + w^*_{bl}$ and $r^*_l = r_f \leq r^*_b$. Hence, by applying for 3PL financing, the retailer benefits from not only a lower purchasing cost but also a lower financing cost. However, when the retailer is extremely cash-strapped with $y \leq 300$, the purchasing cost becomes very high and 3PL financing is dominated by bank financing from its perspective. To figure out whether the retailer can withstand a higher purchasing cost while still obtaining a higher profit under 3PL financing, as shown in Proposition 7, we draw the changes of π_i and $w^*_{is} + w^*_{il}$ in Figure 4 for $y \in [300, 400]$. From Figure 4, we can observe that when $340 < y < 382$, even though the purchasing cost under 3PL financing is higher than that under bank financing, the retailer obtains a higher profit under 3PL financing, which is consistent with the result in Proposition 7.

7 Extensions

7.1 Model when 3PL is Capital Constrained

In practice, some 3PLs may not or only partially use their own working capital in the financing services, especially those expanding their business with no spare money to lend. Nevertheless, to support the development of the financing service, some 3PLs use short-term bank loans or bonds in the financial markets for money. For example, UPS maintains credit of billions of USD with banks (UPS 2019);

Eternal Asia obtains tens of billions of CNY from banks to support its operations, including its supply chain finance business (Eternal Asia 2018). Therefore, in this section, we consider the case of a 3PL borrowing money from a bank to invest in the financing services.

At time $t = 0$, we assume the 3PL's working capital is Y. When the retailer chooses financing from a capital-constrained 3PL, the 3PL obtains money $\min(y, (w_{ls} + w_{ll})q_l)$ from the retailer and spends $(w_{ls} + c_l)q_l$ on the purchasing and delivering processes. If $Y + \min(y, (w_{ls} + w_{ll})q_l) < (w_{ls} + c_l)q_l$, the 3PL needs to borrow from the bank and we define the money borrowed as $B_l := [(w_{ls} + c_l)q_l - Y - \min(y, (w_{ls} + w_{ll})q_l)]^+$. At the end of the selling season, the retailer repays 3PL financing at interest rate r_l and the 3PL obtains $\min\{p\min(q_l, D) + S, [(w_{ls} + w_{ll})q_l - y]^+(1 + r_l)\}$. Consequently, the 3PL's capital level at the end of $t = 1$ equals $L_l := [Y + \min(y, (w_{ls} + w_{ll})q_l) - (w_{ls} + c_l)q_l]^+(1 + r_f) + \min\{p\min(q_l, D) + S, [(w_{ls} + w_{ll})q_l - y]^+(1 + r_l)\}$. If the 3PL borrows from the bank at $t = 0$, the 3PL needs to repay the bank loan with interest rate. Similar to our analysis of when the retailer is financed by the bank in Section 4, we assume the 3PL is a limited liability company and repays the bank loan with its capital at the end of the selling season. Hence, the money the 3PL can repay the bank equals $\min(L_l, B_l(1 + r_b))$, where r_b is the bank's interest rate. Consequently, the 3PL's final expected profit is

$$\Gamma_l(w_{ll}, r_l) = \mathbb{E}[L_l - \min(L_l, B_l(1 + r_b))] - Y(1 + r_f). \tag{21}$$

Note that the bank loan is competitively priced, which means $\min(L_l, B_l(1 + r_b)) = B_l(1 + r_f)$. Substituting it in Equation (21), we can rewrite the 3PL's expected profit as

$$\Gamma_l(w_{ll}, r_l) = \mathbb{E}[L_l - B_l(1 + r_f)] - Y(1 + r_f). \tag{22}$$

Proposition 9 *The 3PL's decisions on transportation price and financing interest rate are unaffected by its working capital level.*

Proposition 9 reveals that the 3PL's decisions are independent of its capital level. Consequently, the retailer and supplier's decisions are also independent of the 3PL's capital status. Recall that in Section 4, when bank financing is adopted by the retailer, we arrive at a similar conclusion that all the supply chain members' decisions are unaffected by the retailer's capital level and bank's interest rate. These conclusions are based on the assumptions that the capital market is perfect and bank loans are competitively priced. These assumptions may not hold in practice. However, from the 3PL's perspective, big 3PLs such as Eternal Asia and UPS can borrow from banks at a very low cost because of their low default risks. Therefore, Proposition 9's conclusion is not significantly affected by the assumptions. If the retailer borrows from the bank at a higher interest rate, from its perspective, bank financing is less attractive and 3PL financing will be preferred by the supply chain members, so our conclusions in Section 6 are also not significantly affected by the assumptions.

7.2 Model when the 3PL Functions as the Channel Leader

In the previous analyses, we assumed that the supplier functions as the supply chain leader and the 3PL as the sub-leader. Under this setting, the supplier determines the wholesale price first and subsequently, the 3PL determines the transportation price and financing interest rate. It is worth noting that a change in leadership related to the sequence of events may have significant impacts on the outcomes of the Stackelberg game. In Zhou et al. (2020), when a supplier or 3PL guarantees a retailer's bank loans, the result shows that supplier leadership is usually preferable for both the retailer and supply chain. Conversely, when a 3PL grants payment delay to a retailer, Chen et al. (2019a) conclude that 3PL leadership outperforms supplier leadership.

In this section, we explore the impact of leadership changes on supply chain performance. In particular, we briefly study the design of a 3PL financing contract when the 3PL functions as the leader and compare these results with those obtained for supplier leadership. To distinguish the 3PL leadership notations and results from those of supplier leadership, we use variables with $\sim$, $\tilde{q}_s$, for example, to represent the variables under the 3PL leadership scenario. Furthermore, to simplify the analysis, we consider only the case where the retailer is capital constrained and requires 3PL financing.

The decision sequence is as follows: (1) the 3PL determines the transportation price $\tilde{w}_{ll}$ and financing interest rate $\tilde{r}_l$; (2) the supplier determines the wholesale price $\tilde{w}_{ls}$; (3) given the 3PL and supplier's decisions, the retailer determines the order quantity $\tilde{q}_l$. All the supply chain members' objective functions are the same as in the supplier leadership scenario.

Proposition 10 *When the 3PL functions as the supply chain leader,*

(i) the retailer's optimal order quantity $\tilde{q}_l = \bar{F}^{-1}\left((\tilde{w}_{ls}+\tilde{w}_{ll})(1+\tilde{r}_l)\bar{F}(\tilde{z}_l)/p\right)$*, where*

$$\tilde{z}_l = \frac{[(\tilde{w}_{ls}+\tilde{w}_{ll})\tilde{q}_l - y](1+\tilde{r}_l) - S}{p}; \tag{23}$$

(ii) the supplier's optimal wholesale price $\tilde{w}_{ls} = p\bar{F}(\tilde{q}_l)/[(1+\tilde{r}_l)\bar{F}(\tilde{z}_l)] - \tilde{w}_{ll}$*, and* $\tilde{q}_l$ *uniquely solves*

$$\frac{p\bar{F}(\tilde{q}_l)}{\bar{F}(\tilde{z}_l)}\frac{1-\tilde{q}_l h(\tilde{q}_l)}{1-\tilde{\delta}_l} - (\tilde{w}_{ll}+c_s)(1+\tilde{r}_l) = 0, \tag{24}$$

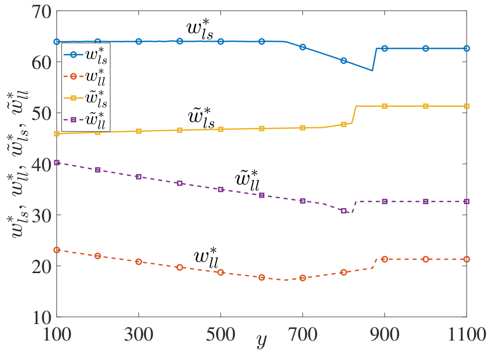
where $\tilde{\delta}_l = (\tilde{w}_{ls}+\tilde{w}_{ll})(1+\tilde{r}_l)\tilde{q}_l h(\tilde{z}_l)/p$.

Comparing Proposition 10 (i) with Proposition 2, we find that the retailer's optimal response functions under 3PL leadership and supplier leadership are unchanged. This means that when the relevant costs (wholesale price, transportation price, and interest rate) are unchanged under the two channel structures, the order quantities are equal, and the change of leadership has no impact on the retailer's decision. However, compared with the results in Subsection 5.3, Proposition 10 (ii) indicates that the supplier's decision is influenced by the change of leadership, and hence the wholesale prices under the two channel structures will differ. Consequently, the retailer's decision might also differ.

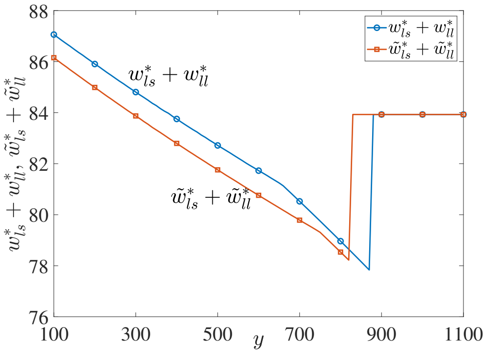
Since the 3PL's problem is very complex when it is the channel leader, the theoretical analysis of the 3PL's optimal decisions on transportation price and financing interest rate is hindered. To conduct the analysis, we run a numerical experiment to explore all the supply chain parties' optimal decisions and returns. Furthermore, we compare these results with those obtained in Section 6 with the supplier functioning as the channel leader. The parameter settings here are the same as those in the numerical experiment in Subsection 6.3. We also consider some other parameter settings, for instance, the production cost is less than the transportation cost (i.e., $c_s \leq c_l$) or D follows a truncated normal distribution, in the numerical experiment and obtain similar results.

The result shows that the retailer requires financing when its capital $y \leq 990$ under both supplier leadership and 3PL leadership. Additionally, the 3PL should set the interest rate $\tilde{r}_l^* = r_f$, which is the same as that in the supplier leadership scenario. Other results, such as the optimal wholesale price, transportation price, order quantity, and all members' expected profits are shown in Figure 5. From Figures 5(a), 5(d), and 5(e), we can observe that a first-mover advantage exists in the Stackelberg game. When the supplier (3PL) is the channel leader and moves first, it charges a higher wholesale (transportation) price, and obtains a higher marginal profit and final expected profit than when it is the subleader. Figures 5(b), 5(c), and 5(f) show that, when the retailer suffers severe capital constraints ($y \leq 820$), comparing with the scenario under supplier leadership, the retailer is better off and orders more because of a lower unit purchasing cost under 3PL leadership. The underlying reason is that, when functioning as the leader of the supply chain, the 3PL has a first-mover advantage to capture more profit, which stimulates it to share more risk with the retailer by providing financing services and undertaking a higher default risk. As a consequence, the retailer has an incentive to order more products which benefits itself and the supply chain. This conclusion is similar to the result of Chen et al. (2019a) but contrary to that of Zhou et al. (2020). Zhou et al. (2020) examine the supplier and 3PL guaranteed bank financing problems, and assume that the average market demand is a linear function of the selling price, which is influenced by the wholesale and transportation prices. By contrast, in our paper, the random demand is independent of those prices as the selling price is constant.

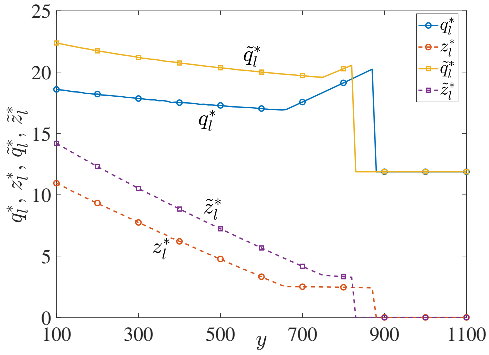
When 3PL is the channel leader, at point $y = 820$, the unit purchasing cost $\tilde{w}_{ls}^* + \tilde{w}_{ll}^* < w_{ls}^* + w_{ll}^*$ is low enough. In the interval of [830,870], both the 3PL and supplier prefer a higher marginal profit and the order quantity is lower under 3PL leadership. By contrast, under supplier leadership, both of them prefer a higher order quantity and the marginal profit is lower. Thus, $\tilde{q}_l^* < q_l^*$ and the retailer is better off under supplier leadership. When $y \geq 880$, $z_l^* = \tilde{z}_l^* = 0$, the unit purchasing cost, optimal order quantity, and retailer's expected profit are not influenced by the change of leadership. Please note that, because of the retailer's collateral assets S, $z_l^* = \tilde{z}_l^* = 0$ do not mean that the retailer has



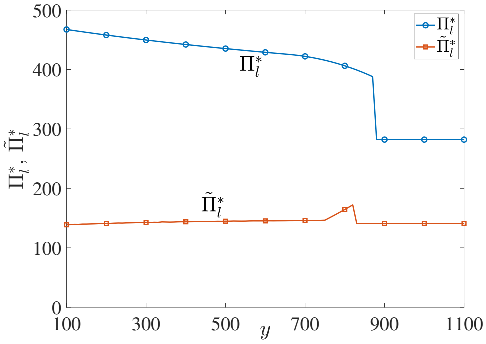
(a) Optimal wholesale and transportation prices



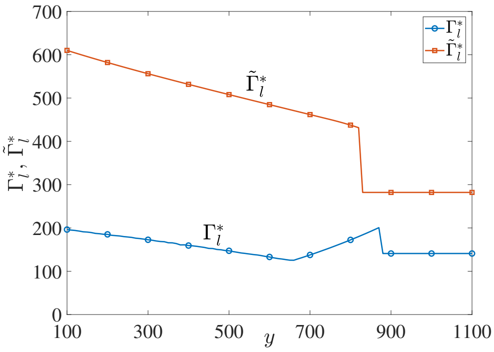
(b) Retailer's unit purchasing prices



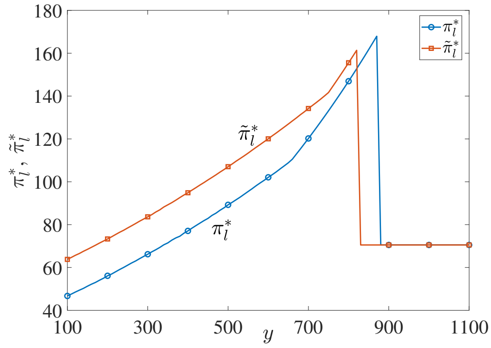
(c) Optimal order quantities and bankruptcy thresholds



(d) Supplier's optimal expected profits



(e) 3PL's optimal expected profits



(f) Retailer's optimal expected profits

Figure 5: Comparison of results under 3PL financing when the supplier or the 3PL functions as the channel leader.

sufficient working capital and does not need financing. Financing is needed as long as $y \leq 990$ under both cases.

8 Conclusions and Future Research

In this study, we focus on the interaction between operational decisions and short-term financing by studying 3PL financing in a three-tier supply chain. Through theoretical analysis, we attempt to understand why 3PLs are willing to finance retail inventories in practice. We derive the Stackelberg equilibria of the optimal 3PL financing scheme design and compare 3PL financing with bank financing from the perspectives of the relevant members and the overall supply chain. Additionally, in the extensions, we consider the cases of a capital-constrained 3PL borrowing from banks, and a 3PL functioning as the supply chain leader. The primary results are summarized as follows.

First, under the framework where the supplier is the leader, 3PL is the sub-leader, and retailer is the follower, our results indicate that the 3PL should set the financing interest rate as low as the risk-free rate to alleviate the retailer's capital pressure. Consequently, the retailer's financing cost is reduced and it will order more products. Then, we explore the conditions under which all the members benefit from 3PL financing to ensure this financing service's sustainability.

Second, compared with bank financing, 3PL financing leads to greater supply chain efficiency by encouraging the retailer to order more. Unfortunately, neither bank financing nor 3PL financing can coordinate the supply chain. From the supplier's perspective, 3PL financing always outperforms bank financing. Given the supplier's optimal wholesale price, the 3PL and retailer also prefer 3PL financing. However, without the influence of the supplier's decision, the 3PL and retailer might prefer bank financing over 3PL financing in certain cases. Furthermore, we demonstrate that under 3PL financing, all channel participants can achieve Pareto improvements when the wholesale price and the sum of the wholesale and transportation prices are lower than certain thresholds.

Finally, when the 3PL is capital constrained and bank loans are available, our analysis reveals that the design of the 3PL financing scheme is independent of the 3PL's capital level. However, when the 3PL functions as the supply chain leader instead of the supplier, the 3PL financing scheme design differs. We analyze the retailer and supplier's decisions under 3PL leadership, and conduct a numerical study to compare 3PL leadership with supplier leadership. The results indicate that when the 3PL functions as the channel leader, the 3PL financing interest rate equals the risk-free interest rate. In addition, the retailer has a lower purchase cost, a larger order size, and a higher bankruptcy risk when it has severe capital constraints. For the supplier and 3PL, there exists a first-mover advantage in the Stackelberg game. That is, the player can obtain more profit when it functions as the leader rather than the sub-leader.

Future research can be conducted in various contexts. In this study, we consider a case of a 3PL offering financing services to only one retailer. However, 3PLs often serve multiple customers in the industry simultaneously, which may help the 3PLs make risk diversification strategies and reduce default risks. Therefore, one can explore the 3PL financing scheme when multiple capital-constrained retailers exist in a supply chain. In addition, the risk-neutral assumption has been well taken in the literature and this paper, but decisions makers, especially entrepreneurs or the owners of small firms, may be risk-averse or risk-seeking in practice. Although we believe the advantage of 3PL financing still exists when the risk-neutral assumption is violated, an extension exploring the influences of risk preferences on the financing and operational decisions will hopefully provide us with additional managerial insights.

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Appendix

Proof of Proposition 1. We have shown the decision processes of the bank, retailer, and 3PL from Subsection 4.1 to 4.3. Next, we will only prove the supplier's optimal wholesale price w_{bs}^*.

Taking the first- and second-order derivatives of $\Pi_b(q_b)$ in Equation (9) with respect to q_b,

$$\frac{d\Pi_b(q_b)}{dq_b} = p\bar{F}(q_b)\left\{[1-q_b h(q_b)]^2 - q_b[h(q_b)+q_b h'(q_b)]\right\} - (c_s+c_l)(1+r_f),$$

$$\begin{aligned}\frac{d^2\Pi_b(q_b)}{d{q_b}^2} =& -pf(q_b)[1-q_b h(q_b)]^2 - 3p\bar{F}(q_b)[h(q_b)+q_b h'(q_b)][1-q_b h(q_b)] \\ & - pq_b\bar{F}(q_b)[2h'(q_b)+q_b h''(q_b)].\end{aligned}$$

As the constraint $q_b < q_\alpha$ holds and $h(q_b)$ is an increasing and concave function, we know $\frac{d^2\Pi_b(q_b)}{d{q_b}^2} < 0$, which means $\Pi_b(q_b)$ is a concave function in the interval $[0, q_\alpha]$. Meanwhile, as $\frac{d\Pi_b(q_b)}{dq_b}|_{q_b=0} = p-(c_s+c_l)(1+r_f) > 0$ and $\frac{d\Pi_b(q_b)}{dq_b}|_{q_b=q_\alpha} = -p\bar{F}(q_\alpha)q_\alpha[h(q_\alpha)+q_\alpha h'(q_\alpha)] - (c_s+c_l)(1+r_f) < 0$, to the supplier, the optimal order quantity q_b^* satisfies the first-order condition $p\bar{F}(q_b^*)\left\{[1-q_b^* h(q_b^*)]^2 - q_b^*\left[h(q_b^*)+q_b^* h'(q_b^*)\right]\right\} - (c_s+c_l)(1+r_f) = 0$. Please note that q_b^* is also the retailer's final order quantity. Combined with Equation (7), the supplier's optimal wholesale price w_{bs}^* is determined by $w_{bs}^* = \frac{p\bar{F}(q_b^*)[1-q_b^* h(q_b^*)]}{1+r_f} - c_l$.

□

Proof of Lemma 1. First, we assume that $z_l \geq q_l$. Combined with the definition, we have relationship (i) $pz_l = \{[(w_{ls}+w_{ll})q_l - y]^+(1+r_l) - S\}^+ \geq pq_l$. However, from $(w_{ls}+w_{ll})(1+r_l) \leq p$, we know relationship (ii) $\{[(w_{ls}+w_{ll})q_l - y]^+(1+r_l) - S\}^+ < pq_l$ always holds, which is contradictory to relationship (i). Therefore, we have $z_l < q_l$.

□

Proof of Proposition 2. The result of Equation (13b) is obvious when the retailer orders with all its working capital. Then we will only show the proofs of Equations (13a) and (13c).

Equations (12a) and (12c) can be rewritten as

$$\pi_l(q_l) =$$

$$\begin{cases} \displaystyle\int_{z_l}^{q_l} pxf(x)\,dx + \int_{q_l}^{N} pq_l f(x)\,dx - \int_{z_l}^{N} pz_l f(x)\,dx - y(1+r_f) - S, & \text{if } (w_{ls}+w_{ll})q_l > y + \dfrac{S}{1+r_l}, \\ \displaystyle\int_{0}^{q_l} pxf(x)\,dx + \int_{q_l}^{N} pq_l f(x)\,dx - (w_{ls}+w_{ll})q_l(1+r_l) + y(r_l - r_f), & \text{if } y < (w_{ls}+w_{ll})q_l \leq y + \dfrac{S}{1+r_l}, \\ \displaystyle\int_{0}^{q_l} pxf(x)\,dx + \int_{q_l}^{N} pq_l f(x)\,dx - (w_{ls}+w_{ll})q_l(1+r_f), & \text{if } (w_{ls}+w_{ll})q_l < y. \end{cases}$$

By taking the first- and second-order derivatives of $\pi_l(q_l)$ with respective to q_l, we have

$$\frac{d\pi_l(q_l)}{dq_l} = \begin{cases} p\bar{F}(q_l) - (w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l), & \text{if } (w_{ls}+w_{ll})q_l > y + \dfrac{S}{1+r_l}, \\ p\bar{F}(q_l) - (w_{ls}+w_{ll})(1+r_l), & \text{if } y < (w_{ls}+w_{ll})q_l \le y + \dfrac{S}{1+r_l}, \\ p\bar{F}(q_l) - (w_{ls}+w_{ll})(1+r_f), & \text{if } (w_{ls}+w_{ll})q_l < y, \end{cases}$$

and

$$\frac{d^2\pi_l(q_l)}{dq_l^2} = \begin{cases} -pf(q_l) + f(z_l)\dfrac{(w_{ls}+w_{ll})^2(1+r_l)^2}{p}, & \text{if } (w_{ls}+w_{ll})q_l > y + \dfrac{S}{1+r_l}, \\ -pf(q_l), & \text{if } y < (w_{ls}+w_{ll})q_l \le y + \dfrac{S}{1+r_l}, \\ -pf(q_l), & \text{if } (w_{ls}+w_{ll})q_l < y. \end{cases}$$

For the case of $(w_{ls}+w_{ll})q_l > y + \frac{S}{1+r_l}$, when the first-order condition is satisfied, i.e., $\frac{d\pi_l(q_l)}{dq_l} = 0$, we obtain $\frac{d^2\pi_l(q_l)}{dq_l^2} = -\bar{F}(q_l)[ph(q_l) - (w_{ls}+w_{ll})(1+r_l)h(z_l)] < 0$. For the cases of $y < (w_{ls}+w_{ll})q_l \le y + \frac{S}{1+r_l}$ and $(w_{ls}+w_{ll})q_l < y$, $\frac{d^2\pi_l(q_l)}{dq_l^2} < 0$. Thus, we can obtain the optimal solutions from the first-order conditions. Additionally, we can combine the solutions under $(w_{ls}+w_{ll})q_l > y + \frac{S}{1+r_l}$ and $y < (w_{ls}+w_{ll})q_l \le y + \frac{S}{1+r_l}$ into one, which is $q_l = \bar{F}^{-1}\left((w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)/p\right)$ because $z_l > 0$ for $(w_{ls}+w_{ll})q_l > y + \frac{S}{1+r_l}$ and $z = 0$ for $y < (w_{ls}+w_{ll})q_l \le y + \frac{S}{1+r_l}$.

□

Proof of Lemma 2. According to Figure 2, when the retailer has sufficient working capital, we can reorganize Equation (13c) as $\bar{F}(q_l)q_l = \frac{(w_{ls}+w_{ll})q_l(1+r_f)}{p}$. Meanwhile, as $(w_{ls}+w_{ll})q_l < y$, we have $\bar{F}(q_l)q_l < \frac{y(1+r_f)}{p}$, i.e., $q_l \in \Omega_3$.

When the retailer needs to apply for 3PL financing, we can reorganize Equation (13a) as $\bar{F}(q_l)q_l = \frac{(w_{ls}+w_{ll})q_l(1+r_l)\bar{F}(z_l)}{p}$. From the definition, we know $z_l \ge 0$.

(i) When $z_l = 0$ and the retailer needs 3PL financing, $y < (w_{ls}+w_{ll})q_l \le y + \frac{S}{1+r_l}$ and $\frac{y(1+r_l)}{p} < \bar{F}(q_l)q_l \le \frac{y(1+r_l)+S}{p}$, which means $q_1^l < q_l \le {q_1^l}'$ and ${q_1^u}' \le q_l < q_1^u$, where ${q_1^l}'$ and ${q_1^u}'$ are two numbers that solve $\bar{F}(q_l)q_l = \frac{y(1+r_l)+S}{p}$.

(ii) When $z_l > 0$, i.e., $(w_{ls}+w_{ll})q_l > y + \frac{S}{1+r_l}$, based on the definition of z_l, we take the first-order derivative of $(w_{ls}+w_{ll})q_l$ with respect to z_l and obtain $\frac{d(w_{ls}+w_{ll})q_l}{dz_l} = \frac{p}{1+r_l}$. Next, we take a derivative from both sides of the equation $\bar{F}(q_l)q_l = \frac{(w_{ls}+w_{ll})q_l(1+r_l)\bar{F}(z_l)}{p}$ with respect to z_l and obtain

$$\begin{aligned} \frac{d\bar{F}(q_l)q_l}{dz_l} &= \frac{d(w_{ls}+w_{ll})q_l}{dz_l}\frac{(1+r_l)\bar{F}(z_l)}{p} - \frac{(w_{ls}+w_{ll})q_l(1+r_l)}{p}f(z_l) \\ &= \bar{F}(z_l)\left[1-\delta_l\right], \end{aligned}$$

where $\delta_l = \frac{(w_{ls}+w_{ll})(1+r_l)}{p}q_lh(z_l)$ for $z_l > 0$. As $\bar{F}(q_l)q_l$ is a quasi-concave function with highest value obtained at point q_α, we have

$$\begin{aligned}\bar{F}(q_\alpha)q_\alpha &\geq \bar{F}(q_l)q_l \\ &= \frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l \bar{F}\left(\frac{[(w_{ls}+w_{ll})q_l - y](1+r_l)}{p}\right) \\ &> \frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l \bar{F}\left(\frac{(w_{ls}+w_{ll})(1+r_l)q_l}{p}\right).\end{aligned}$$

It is obvious that $\frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l \leq q_\alpha$. Then

$$\begin{aligned}\frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l h(z_l) &< \frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l h\left(\frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l\right) \\ &\leq q_\alpha h(q_\alpha) \\ &= 1.\end{aligned}$$

Therefore, $\delta_l < 1$ and $\frac{d\bar{F}(q_l)q_l}{dz_l} > 0$, i.e., $\bar{F}(q_l)q_l$ increases in z_l. Then we can prove that when $z_l > 0$, $\bar{F}(q_l)q_l > \frac{y(1+r_l)+S}{p}$ and $q_l \in ({q_1^l}', {q_1^u}')$. Combined with cases (i) and (ii), we conclude that $q_l \in \Omega_1$ when the retailer needs 3PL financing.

Finally, when the retailer spends all its initial capital on purchasing, i.e., $(w_{ls}+w_{ll})q_l = y$, we have $q_l \in \Omega_2$.

□

Proof of Lemma 3. First, we prove the first part of Lemma 3(i), i.e., π_l monotonously decreases in $(w_{ls}+w_{ll})$.

When $(w_{ls}+w_{ll})q_l > y$ and $z_l > 0$, by taking the first-order derivative of π_l in Equation (12a) with respect to $(w_{ls}+w_{ll})$,

$$\frac{d\pi_l}{d(w_{ls}+w_{ll})} = p\bar{F}(q_l)\frac{dq_l}{d(w_{ls}+w_{ll})} - p\bar{F}(z_l)\frac{dz_l}{d(w_{ls}+w_{ll})}.$$

From $p\bar{F}(q_l) = (w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)$ and $pz_l = (w_{ls}+w_{ll})q_l(1+r_l) - y(1+r_l) - S$, we know

$$p\frac{dz_l}{d(w_{ls}+w_{ll})} = q_l(1+r_l) + (w_{ls}+w_{ll})(1+r_l)\frac{dq_l}{d(w_{ls}+w_{ll})}$$

Then

$$\frac{d\pi_l}{d(w_{ls}+w_{ll})} = p\frac{dq_l}{d(w_{ls}+w_{ll})}\left[\bar{F}(q_l) - (w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)\right] - \bar{F}(z_l)q_l(1+r_l) < 0.$$

Similarly, we can obtain the same conclusion when $(w_{ls}+w_{ll})q_l > y$ and $z_l = 0$, $(w_{ls}+w_{ll})q_l = y$, and $(w_{ls}+w_{ll})q_l < y$. Meanwhile, since

$$\begin{aligned}
&\lim_{(w_{ls}+w_{ll})q_l \to y+\frac{S}{1+r_l}} \pi_l\left(q_l|(w_{ls}+w_{ll})q_l > y + S/(1+r_l)\right) \\
&= \lim_{(w_{ls}+w_{ll})q_l \to y+\frac{S}{1+r_l}} \pi_l\left(q_l|(w_{ls}+w_{ll})q_l < y + S/(1+r_l)\right), \\
&\lim_{y \to (w_{ls}+w_{ll})q_l} \pi_l(q_l|(w_{ls}+w_{ll})q_l > y) = \mathbb{E}[p\min(q_l, D)] - y(1+r_f) = \pi_l(q_l|(w_{ls}+w_{ll})q_l = y), \\
&\lim_{y \to (w_{ls}+w_{ll})q_l} \pi_l(q_l|(w_{ls}+w_{ll})q_l < y) = \mathbb{E}[p\min(q_l, D)] - y(1+r_f) = \pi_l(q_l|(w_{ls}+w_{ll})q_l = y),
\end{aligned}$$

we know π_l is continuous. Thus, π_l decreases in $(w_{ls}+w_{ll})$.

Next, we prove q_l monotonously decreases in $(w_{ls}+w_{ll})$. According to Equation (13) and Figure A1, when $(w_{ls}+w_{ll})q_l = y$, $(w_{ls}+w_{ll})q_l < y$, and $(w_{ls}+w_{ll})q_l > y$ and $z_l = 0$, it is obvious that q_l decreases in $(w_{ls}+w_{ll})$. When $(w_{ls}+w_{ll})q_l > y$ and $z_l > 0$, from the definition of z_l we know $\frac{dz_l}{d(w_{ls}+w_{ll})} = \frac{(1+r_l)q_l}{p} + \frac{(w_{ls}+w_{ll})(1+r_l)}{p}\frac{dq_l}{d(w_{ls}+w_{ll})}$. Combined with

$$-pq_l f(q_l)\frac{dq_l}{d(w_{ls}+w_{ll})} = \bar{F}(z_l)(1+r_l)q_l - (w_{ls}+w_{ll})(1+r_l)q_l f(z_l)\frac{dz_l}{d(w_{ls}+w_{ll})}.$$

which is derived from $p\bar{F}(q_l) = (w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)$, we have

$$\frac{dq_l}{d(w_{ls}+w_{ll})} = \frac{(1+r_l)\bar{F}(z_l)}{p\bar{F}(q_l)} \frac{1-\delta_l}{\frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l) - h(q_l)} < 0,$$

The "<" holds because $\delta_l < 1$ and $\frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l) < h(q_l)$. Then the second part of Lemma 3(i) is proved.

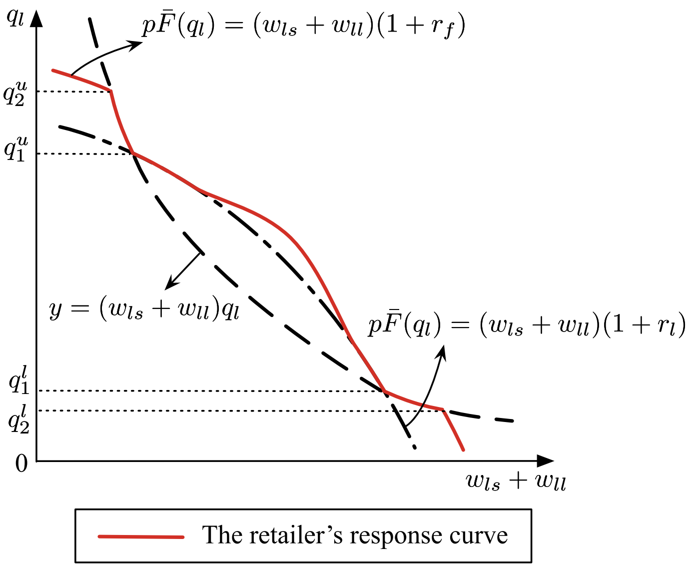


Figure A1: One-to-one mapping between q_l and $w_{ls}+w_{ll}$.

The proofs of Lemma 3(ii) and (iii) are similar to that of (i), so we omit them.

□

Proof of Lemma 4. By taking the first-order derivative of $\Gamma_l(q_l, r_l)$ with respect to q_l, we obtain

$$\frac{\partial \Gamma_l(q_l, r_l)}{\partial q_l} = \begin{cases} \dfrac{p\bar{F}(q_l)[1-q_l h(q_l)]}{1-\delta_l} - (w_{ls}+c_l)(1+r_f), & \text{if } q_l \in \Omega_1 \text{ and } z_l > 0, \quad \text{(A1a)} \\ p\bar{F}(q_l)[1-q_l h(q_l)] - (w_{ls}+c_l)(1+r_f), & \text{if } q_l \in \Omega_1 \text{ and } z_l = 0, \quad \text{(A1b)} \\ -(w_{ls}+c_l)(1+r_f) < 0, & \text{if } q_l \in \Omega_2, \quad \text{(A1c)} \\ p\bar{F}(q_l)[1-q_l h(q_l)] - (w_{ls}+c_l)(1+r_f), & \text{if } q_l \in \Omega_3. \quad \text{(A1d)} \end{cases}$$

From Equations (A1a) to (A1d), we know that, to the 3PL, the optimal $q_l \in [0, q_\alpha)$. For the convenience of expression, we define $\hat{G}(q_l)$ and $\bar{G}(q_l)$ as follows,

$$\begin{cases} \hat{G}(q_l) = \dfrac{p\bar{F}(q_l)[1-q_l h(q_l)]}{1-\delta_l} - (w_{ls}+c_l)(1+r_f), \\ \bar{G}(q_l) = p\bar{F}(q_l)[1-q_l h(q_l)] - (w_{ls}+c_l)(1+r_f). \end{cases}$$

Then, we analyze the monotonicity of $\hat{G}(q_l)$.

$$\begin{aligned} \frac{\partial}{\partial q_l} \ln\left\{\frac{p\bar{F}(q_l)[1-q_l h(q_l)]}{1-\delta_l}\right\} &= \frac{\partial}{\partial q_l}\ln[p\bar{F}(q_l)] + \frac{\partial}{\partial q_l}\ln[1-q_l h(q_l)] - \frac{\partial}{\partial q_l}\ln(1-\delta_l) \\ &= -\frac{f(q_l)}{\bar{F}(q_l)} - \frac{h(q_l)+q_l h'(q_l)}{1-q_l h(q_l)} + \frac{1}{1-\delta_l}\frac{d\delta_l}{dq_l}, \end{aligned}$$

where

$$\frac{d\delta_l}{dq_l} = \frac{1+r_l}{p} q_l h(z_l)\frac{dw_{ll}}{dq_l} + \frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l) + \frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l h'(z_l)\frac{dz_l}{dq_l}.$$

From $pz_l = [(w_{ls}+w_{ll})q_l - y](1+r_l) - S$, we know

$$p\frac{dz_l}{dq_l} = q_l(1+r_l)\frac{dw_{ll}}{dq_l} + (w_{ls}+w_{ll})(1+r_l). \quad \text{(A2)}$$

Therefore, we can rewrite $\frac{d\delta_l}{dq_l}$ as

$$\frac{d\delta_l}{dq_l} = \left[\frac{1+r_l}{p} q_l \frac{dw_{ll}}{dq_l} + \frac{(w_{ls}+w_{ll})(1+r_l)}{p}\right]\left[h(z_l) + \frac{(w_{ls}+w_{ll})(1+r_l)}{p} q_l h'(z_l)\right]. \quad \text{(A3)}$$

In addition, according to $p\bar{F}(q_l) = (w_{ls}+w_{ll})(1+r_l)\bar{F}(z_l)$, we know

$$-pf(q_l) = (1+r_l)\bar{F}(z_l)\frac{dw_{ll}}{dq_l} = (w_{ls}+w_{ll})(1+r_l)f(z_l)\frac{dz_l}{dq_l}. \quad \text{(A4)}$$

Then, combined with Equations (A2) and (A4), we have

$$\frac{dw_{ll}}{dq_l} = \frac{p\left[\frac{\bar{F}(q_l)}{\bar{F}(z_l)}\right]^2 f(z_l) - pf(q_l))}{(1+r_l)[\bar{F}(z_l) - q_l\bar{F}(q_l)h(z_l)]}.$$

By plugging it into Equation (A3) and after simplification,

$$\frac{d\delta_l}{dq_l} = \frac{(w_{ls}+w_{ll})(1+r_l)}{p}\frac{1-q_l h(q_l)}{1-\delta_l}\left[h(z_l)+\frac{(w_{ls}+w_{ll})(1+r_l)}{p}q_l h'(z_l)\right].$$

Finally, we find that $\frac{\partial}{\partial q_l}\ln\left\{\frac{p\bar{F}(q_l)[1-q_l h(q_l)]}{1-\delta_l}\right\}<0$ as

$$\begin{aligned}
&\frac{\partial}{\partial q_l}\ln\left\{\frac{p\bar{F}(q_l)[1-q_l h(q_l)]}{1-\delta_l}\right\} \qquad \text{(A5)}\\
=&-\frac{f(q_l)}{\bar{F}(q_l)}-\frac{h(q_l)+q_l h'(q_l)}{1-q_l h(q_l)}+\frac{1}{1-\delta_l}\frac{d\delta_l}{dq_l}\\
<&-\frac{f(q_l)}{\bar{F}(q_l)}-\frac{h(q_l)+q_l h'(q_l)-\frac{d\delta_l}{dq_l}}{1-q_l h(q_l)}\\
=&-\frac{f(q_l)}{\bar{F}(q_l)}-\frac{h(q_l)+q_l h'(q_l)-\frac{(w_{ls}+w_{ll})(1+r_l)}{p}\frac{1-q_l h(q_l)}{1-\delta_l}\left[h(z_l)+\frac{(w_{ls}+w_{ll})(1+r_l)}{p}q_l h'(z_l)\right]}{1-q_l h(q_l)}\\
=&-\frac{f(q_l)}{\bar{F}(q_l)}-\frac{h(q_l)-\frac{(w_{ls}+w_{ll})(1+r_l)}{p}\frac{1-q_l h(q_l)}{1-\delta_l}h(z_l)+q_l h'(q_l)-\left[\frac{(w_{ls}+w_{ll})(1+r_l)}{p}\right]^2\frac{1-q_l h(q_l)}{1-\delta_l}q_l h'(z_l)}{1-q_l h(q_l)}\\
<&0.
\end{aligned}$$

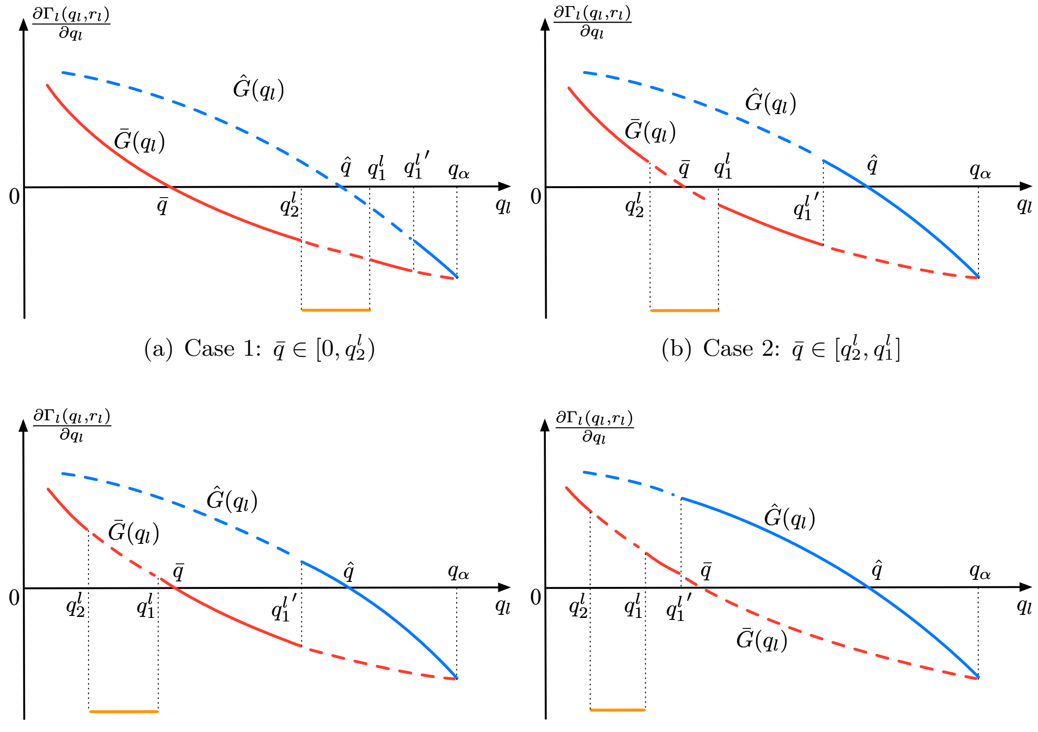
The first "$<$" holds because $1-q_l h(q_l)<1-\delta_l$; the second "$<$" holds because $h(z_l)<h(q_l)$, $h'(z_l)<h'(q_l)$, $\frac{(w_{ls}+w_{ll})(1+r_l)}{p}<1$, and $\frac{1-q_l h(q_l)}{1-\delta_l}<1$. Thus, we can conclude that $\hat{G}(q_l)$ decreases in q_l, which means $\frac{\partial^2\Gamma_l(q_l,r_l)}{\partial q_l{}^2}<0$. Consequently, the optimal q_l is achieved at $\hat{q}$ for $q_l\in\Omega_1$ and $z_l>0$.

In terms of $\bar{G}(q_l)$, we can easily prove that it decreases in q_l. Thus, the optimal q_l is achieved at $\bar{q}$ for $q_l\in\Omega_1$ and $z_l>0$, $q_l\in\Omega_2$. However, note that $\hat{q}$ and $\bar{q}$ may be infeasible as we have constraints on q_l in (A1a), (A1b), and (A1d). In Figure A2, we show the four possible cases about the shape of $\frac{\partial\Gamma_l(q_l,r_l)}{\partial q_l}$, which determine the feasibility of $\hat{q}$ and $\bar{q}$. It is worth noting that if $q_l\in(q_1^l,q_1^{l\prime}]$, the retailer applies for 3PL financing but $z_l=0$; if $q_l\in(q_1^{l\prime},q_\alpha)$, the retailer applies for 3PL financing and $z_l>0$.

According to the definitions, when $q_l=0$ or q_α, $\hat{G}(q_l)=\bar{G}(q_l)$; when $q_l\in(0,q_\alpha)$, $\hat{G}(q_l)>\bar{G}(q_l)$. In case 1 of Figure A2, $\bar{G}(q_2^l)<0$ and $\bar{q}\in[0,q_2^l)$. If $\hat{G}(q_1^l)\le 0$, the optimal $q_l=\bar{q}$; otherwise, the optimal $q_l=\arg\max\{\Gamma_l(\bar{q},r_l),\Gamma_l(\hat{q},r_l)\}$. In case 2, $\bar{G}(q_2^l)\ge 0$,$\bar{G}(q_1^l)\le 0$, and $\bar{q}\in[q_2^l,q_1^l]$. If $\hat{G}(q_1^l)\le 0$, the optimal $q_l=q_2^l$; otherwise, the optimal $q_l=\arg\max\{\Gamma_l(\bar{q},r_l),\Gamma_l(\hat{q},r_l)\}$. In case 3, if $\hat{G}(q_1^{l\prime})<0$, $q_l=\arg\max\left\{\Gamma_l(q_2^l,r_l),\Gamma_l(\bar{q},r_l)\right\}$; if $\hat{G}(q_1^{l\prime})\ge 0$, $q_l=\arg\max\left\{\Gamma_l(q_2^l,r_l),\Gamma_l(\bar{q},r_l),\Gamma_l(\hat{q},r_l)\right\}$. In case 4, $q_l=\arg\max\left\{\Gamma_l(q_2^l,r_l),\Gamma_l(\hat{q},r_l)\right\}$.

Based on the above analyses, we know $\bar{q}$, $\hat{q}$, and q_2^l are potential optimal solutions. $\bar{q}$ is feasible only if $\bar{q}\in[0,q_2^l)\cup(q_1^l,q_1^{l\prime}]$, and $\hat{q}$ is feasible only if $\hat{q}\in(q_1^{l\prime},q_\alpha)$.

Next, we will show that q_2^l can be ignored as the optimal solution in the analysis. At point q_2^l, the retailer spends all its initial capital on purchasing without applying for 3PL financing. Then its profit is $\Gamma_l(q_2^l,r_l)=(w_{ll}-c_l)q_2^l(1+r_f)$, which is independent of r_l. Now, we can set $r_l=r_f$, which leads to $q_2^l=q_1^l$. Then $\frac{\partial\Gamma_l(q_l,r_l)}{\partial q_l}$ is a continues function in $[0,q_1^{l\prime}]$. If $\bar{q}\in[0,q_1^{l\prime}]$, $\bar{q}$ is feasible and not worse



(a) Case 1: $\bar{q} \in [0, q_2^l)$

(b) Case 2: $\bar{q} \in [q_2^l, q_1^l]$

(c) Case 3: $\bar{q} \in (q_1^l, {q_1^l}']$

(d) Case 4: $\bar{q} \in ({q_1^l}', q_\alpha)$

Figure A2: Four possible shapes of $\frac{\partial \Gamma_l(q_l,r_l)}{\partial q_l}$

than q_2^l; if $\bar{q} \in ({q_1^l}', q_\alpha)$, $\hat{q}$ is feasible and not worse than q_2^l. Finally, we know q_2^l is not the optimal solution when $r_l = r_f$. As $\Gamma_l(q_l, r_l = r_l^*) \geq \Gamma_l(q_l, r_l = r_f)$, we have the optimal $q_l \in \{\bar{q}, \hat{q}\}$.

□

Proof of Proposition 3. For the first part of Proposition 3, as the 3PL's expected profit is independent of r_l when $q_l \in [0, {q_1^l}']$, we only focus on the optimization of r_l for $q_l \in ({q_1^l}', q_\alpha)$. Next, we prove that $\Gamma(q_l, r_l)$ decreases in r_l in this interval.

Based on Equations (13a) and (17), we define

$$V^1 := \bar{F}(q_l)[1 - q_l h(q_l)] - \frac{(w_{ls} + c_l)(1 + r_f)}{p}(1 - \delta_l) = 0, \tag{A6}$$

$$V^2 := \bar{F}(q_l) - \frac{(w_{ls} + w_{ll})(1 + r_l)}{p}\bar{F}(z_l) = 0. \tag{A7}$$

By taking the first-order derivatives of V^1 and V^2 with respect to q_l, w_{ll}, and r_l, we obtain

$$
\begin{aligned}
V^1_{q_l} &= -\bar{F}(q_l)\left[\begin{aligned}&q_l\left\{h'(q_l) - \frac{1-q_l h(q_l)}{1-\delta_l}\left[\frac{(w_{ls}+w_{ll})(1+r_l)}{p}\right]^2 h'(z_l)\right\}\\ &+ h(q_l) - \frac{1-q_l h(q_l)}{1-\delta_l}\frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l) + h(q_l)[1-q_l h(q_l)]\end{aligned}\right] < 0,\\
V^1_{w_{ll}} &= \frac{\bar{F}(q_l)[1-q_l h(q_l)](1+r_l)q_l}{p(1-\delta_l)}\left[h(z_l) + \frac{(w_{ls}+w_{ll})(1+r_l)q_l h'(z_l)}{p}\right] > 0,\\
V^1_{r_l} &= \frac{\bar{F}(q_l)[1-q_l h(q_l)](w_{ls}+w_{ll})q_l}{p(1-\delta_l)}\left[h(z_l) + \frac{[(w_{ls}+w_{ll})q_l - y](1+r_l)}{p}h'(z_l)\right] > 0,\\
V^2_{q_l} &= -\bar{F}(q_l)\left[h(q_l) - \frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l)\right] < 0,\\
V^2_{w_{ll}} &= -\frac{1+r+l}{p}\bar{F}(z_l)(1-\delta_l) < 0,\\
V^2_{r_l} &= -\frac{w_{ls}+w_{ll}}{p}\bar{F}(z_l)\left\{1 - \frac{[(w_{ls}+w_{ll})q_l - y](1+r_l)}{p}h(z_l)\right\} < 0.
\end{aligned}
$$

Meanwhile, we also define $W := V^1_{q_l}V^2_{w_{ll}} - V^1_{w_{ll}}V^2_{q_l}$, $W_{w_{ll}} := V^1_{r_l}V^2_{q_l} - V^1_{q_l}V^2_{r_l}$, and $W_{q_l} := V^1_{w_{ll}}V^2_{r_l} - V^1_{r_l}V^2_{w_{ll}}$. According to the definitions of $V^i_{q_l}$, $V^i_{w_{ll}}$, $V^i_{r_l}$, $i = 1, 2$, we can figure out that $W > 0$ and $W_{w_{ll}} < 0$. W_{q_l} can be rewritten as $W_{q_{ll}} = -\frac{[\bar{F}(q_l)]^2 q_l y(1+r_l)[1-q_l h(q_l)]}{p^2(1-\delta_l)}[h^2(z_l) + h'(z_l)]$, which means $W_{q_{ll}} < 0$.

Based on Equations (A6) and (A7), we calculate the total derivatives of V^1 and V^2 and obtain $V^1_{q_l}dq_l + V^1_{w_{ll}}dw_{ll} + V^1_{r_l}dr_l = 0$ and $V^2_{q_l}dq_l + V^2_{w_{ll}}dw_{ll} + V^2_{r_l}dr_l = 0$. By reorganizing these two equations, we get

$$
\frac{dq_l}{dr_l} = \frac{V^1_{w_{ll}}V^2_{r_l} - V^1_{r_l}V^2_{w_{ll}}}{V^1_{q_l}V^2_{w_{ll}} - V^1_{w_{ll}}V^2_{q_l}} = \frac{W_{q_l}}{W} < 0, \quad \frac{dw_{ll}}{dr_l} = \frac{V^1_{r_l}V^2_{q_l} - V^1_{q_l}V^2_{r_l}}{V^1_{q_l}V^2_{w_{ll}} - V^1_{w_{ll}}V^2_{q_l}} = \frac{W_{w_{ll}}}{W} < 0.
$$

Next, based on $z_l = \frac{(w_{ls}+w_{ll})q_l(1+r_l) - y(1+r_l) - S}{p}$, we calculate the first-order derivative of z_l with respect to r_l and obtain

$$
\begin{aligned}
\frac{dz_l}{dr_l} &= \frac{1}{p}\left[\frac{dw_{ll}}{dr_l}q_l(1+r_l) + \frac{dq_l}{dr_l}(w_{ls}+w_{ll})(1+r_l) + (w_{ls}+w_{ll})q_l - y\right]\\
&= \frac{1}{pW}\left[V^2_{q_l}\frac{\bar{F}(q_l)[1-q_l h(q_l)]q_l(1+r_l)yh(z_l)}{p(1-\delta_l)} + V^1_{q_l}\frac{\bar{F}(z_l)y(1+r_l)}{p} + (w_{ls}+w_{ll})(1+r_l)W_{q_l}\right].
\end{aligned}
$$

Finally, we calculate the first-order derivative of $\Gamma_l(q_l, r_l)$ with respect to r_l and obtain $\frac{\partial \Gamma_l(q_l,r_l)}{\partial r_l} = p\bar{F}(z_l)\frac{dz_l}{dr_l} - (w_{ls}+c_l)(1+r_f)\frac{dq_l}{dr_l}$. By substituting $\frac{dz_l}{dr_l}$ into it,

$$
\begin{aligned}
\frac{\partial \Gamma_l(q_l,r_l)}{\partial r_l} =& \frac{\bar{F}(z_l)}{W}\left[V^2_{q_l}\frac{\bar{F}(q_l)[1-q_l h(q_l)]q_l(1+r_l)yh(z_l)}{p(1-\delta_l)} + V^1_{q_l}\frac{\bar{F}(z_l)y(1+r_l)}{p} + (w_{ls}+w_{ll})(1+r_l)W_{q_l}\right]\\
&- (w_{ls}+c_l)(1+r_f)\frac{W_{q_l}}{W}\\
=& \frac{\bar{F}(z_l)}{W}\left[\begin{aligned}&V^2_{q_l}\frac{\bar{F}(q_l)[1-q_l h(q_l)]q_l(1+r_l)yh(z_l)}{p(1-\delta_l)} + V^1_{q_l}\frac{\bar{F}(z_l)y(1+r_l)}{p}\\ &- V^2_{q_l}\frac{(w_{ls}+w_{ll})(1+r_l)q_l}{\bar{F}(q_l)(1-\delta_l)}W_{q_l}\end{aligned}\right].
\end{aligned}
$$

Since $\frac{V^2_{w_{ll}}}{1-\delta_l} = -\frac{(1+r_l)\bar{F}(z_l)}{p(1-\delta_l)}$, we know $V^1_{q_l}\frac{\bar{F}(z_l)y(1+r_l)}{p} = -\frac{V^1_{q_l}V^2_{w_{ll}}y}{1-\delta_l}$. Meanwhile, since

$$\begin{aligned}
&V^2_{q_l}\frac{\bar{F}(q_l)[1-q_lh(q_l)]q_l(1+r_l)yh(z_l)}{p(1-\delta_l)} - V^2_{q_l}\frac{(w_{ls}+w_{ll})(1+r_l)q_l}{\bar{F}(q_l)(1-\delta_l)}W_{q_l}\\
=&V^2_{q_l}\frac{\bar{F}(q_l)[1-q_lh(q_l)]q_l(1+r_l)q_ly}{p(1-\delta_l)}\frac{h(z_l)+\frac{(w_{ls}+w_{ll})(1+r_l)q_lh'(z_l)}{p}}{1-\delta_l}\\
=&V^2_{q_l}V^1_{w_{ll}}\frac{y}{1-\delta_l},
\end{aligned}$$

we finally obtain

$$\frac{\partial\Gamma_l(q_l,r_l)}{\partial r_l} = \frac{\bar{F}(z_l)}{W}\frac{V^2_{q_l}V^1_{w_{ll}} - V^1_{q_l}V^2_{w_{ll}}}{1-\delta_l}y = -\frac{y\bar{F}(z_l)}{1-\delta_l} < 0,$$

which means the 3PL's optimal decision on the financing interest rate $r_l^* = r_f$.

Next, we prove the second part of Proposition 3. When $r_l^* = r_f$, $q_2^l = q_1^l$. At point $q_1^{l\prime}$, we know $p\frac{q_1^{l\prime}\bar{F}(q_1^{l\prime})}{1+r_f} = y + \frac{S}{1+r_f}$. Meanwhile, as $q_1^{l\prime} < q_\alpha$, $q_1^{l\prime}$ increases in y. When $y < \underline{\mathbb{C}}$, $q_1^{l\prime} < \bar{q}$ and $\bar{q} \in (q_1^{l\prime}, q_\alpha)$. In this case, $\bar{q}$ is infeasible and thus the optimal $q_l = \hat{q}$. When $y > \bar{\mathbb{C}}$, $q_1^{l\prime} > \hat{q}$. Then $\bar{q}$ is feasible but $\hat{q}$ is infeasible, and the optimal $q_l = \bar{q}$. When $\underline{\mathbb{C}} \le y \le \bar{\mathbb{C}}$, both $\bar{q}$ and $\hat{q}$ are feasible. Then the optimal $q_l = \arg\max(\Gamma_l(\bar{q}, r_f), \Gamma_l(\hat{q}, r_f))$.

□

Proof of Lemma 5. For Lemma 5(i), we prove it by showing that $\frac{dw_{ls}}{dq_l} < 0$. When $y \ge (w_{ls}+w_{ll})q_l - \frac{S}{1+r_f}$, all members' decisions are the same as that in the case of bank financing. Then we can refer to the analyses in Subsection 4.4 and easily figure out that $\frac{dw_{ls}}{dq_l} < 0$. When $y < (w_{ls}+w_{ll})q_l - \frac{S}{1+r_f}$, the optimal order quantity is $\hat{q}$. From Equation (17), we have

$$\begin{aligned}
&-pf(q_l)[1-q_lh(q_l)] - p\bar{F}(q_l)h(q_l) - p\bar{F}(q_l)q_lh'(q_l)\\
=&(1+r_f)(1-\delta_l)\frac{dw_{ls}}{dq_l} - \frac{(w_{ls}+c_l)(1+r_f)(1+r_l)q_lh(z_l)}{p}\left(\frac{dw_{ls}}{dq_l}+\frac{dw_{ll}}{dq_l}\right)\\
&-(w_{ls}+c_l)(1+r_f)\frac{(w_{ls}+w_{ll})(1+r_l)}{p}h(z_l) - \frac{(w_{ls}+c_l)(1+r_f)(w_{ls}+w_{ll})(1+r_l)q_lh'(z_l)}{p}\frac{dz_l}{dq_l}.
\end{aligned} \tag{A8}$$

To analyze $\frac{dw_{ls}}{dq_l}$ in (A8), we need to know the expressions of $\left(\frac{dw_{ls}}{dq_l}+\frac{dw_{ll}}{dq_l}\right)$ and $\frac{dz_l}{dq_l}$. According to Equation (13a) and the definition of z_l, we have

$$-pf(q_l) = (1+r_l)\bar{F}(z_l)\left(\frac{dw_{ls}}{dq_l}+\frac{dw_{ll}}{dq_l}\right) - (w_{ls}+w_{ll})(1+r_l)f(z_l)\frac{dz_l}{dq_l}$$

and

$$\frac{dz_l}{dq_l} = \frac{q_l(1+r_l)}{p}\left(\frac{dw_{ls}}{dq_l}+\frac{dw_{ll}}{dq_l}\right) + \frac{(w_{ls}+w_{ll})(1+r_l)}{p}.$$

By reorganizing them, we obtain

$$\frac{dw_{ls}}{dq_l}+\frac{dw_{ll}}{dq_l}=\frac{p}{q_l(1+r_l)}\frac{dz_l}{dq_l}-\frac{w_{ls}+w_{ll}}{q_l}, \tag{A9}$$

$$\frac{dz_l}{dq_l}=\frac{\bar{F}(q_l)-f(q_l)q_l}{\bar{F}(z_l)\left[1-\frac{(w_{ls}+w_{ll})(1+r_l)}{p}q_l h(z_l)\right]}. \tag{A10}$$

Then we substitute Equations (A9) and (A10) in (A8) and get

$$\begin{aligned}&(1+r_f)(1-\delta_l)\frac{dw_{ls}}{dq_l}\\ =&-pf(q_l)[1-q_l h(q_l)]+\left\{(w_{ls}+c_l)(1+r_f)h(z_l)\frac{\bar{F}(q_l)[1-q_l h(q_l)]}{\bar{F}(z_l)(1-\delta_l)}-p\bar{F}(q_l)h(q_l)\right\}\\ &+\left\{\frac{(w_{ls}+c_l)(1+r_f)(w_{ls}+w_{ll})(1+r_l)q_l h'(z_l)}{p}\frac{\bar{F}(q_l)[1-q_l h(q_l)]}{\bar{F}(z_l)(1-\delta_l)}-p\bar{F}(q_l)q_l h'(q_l)\right\}.\end{aligned} \tag{A11}$$

The formula in the first braces equals $\frac{[1-q_l h(q_l)]^2}{(1-\delta_l)^2}\frac{\bar{F}(q_l)}{\bar{F}(z_l)}h(z_l)p\bar{F}(q_l)-h(q_l)p\bar{F}(q_l)<0$. The "$<$" holds because $\frac{1-q_l h(q_l)}{1-\delta_l}<1$, $\frac{\bar{F}(q_l)}{\bar{F}(z_l)}<1$, and $h(z_l)<h(q_l)$. The formula in the second braces can be rewritten as

$$\bar{F}(q_l)q_l\left[(w_{ls}+w_{ll})(1+r_l)\frac{\bar{F}(q_l)}{\bar{F}(z_l)}\frac{[1-q_l h(q_l)]^2}{(1-\delta_l)^2}h'(z_l)-ph'(q_l)\right]<0.$$

The "$<$" holds because $(w_{ls}+w_{ll})(1+r_l)\le p$, $\frac{\bar{F}(q_l)}{\bar{F}(z_l)}\frac{[1-q_l h(q_l)]^2}{(1-\delta_l)^2}<1$, and $h'(z_l)<h'(q_l)$. Thus, combined with Equation (A11), we find that $\frac{dw_{ls}}{dq_l}<0$ for $y<(w_{ls}+w_{ll})q_l$. Moreover, from Equations (17) and (18), we know $w_{ls}|_{y\to[(w_{ls}+w_{ll})q_l]^-}=w_{ls}|_{y\to[(w_{ls}+w_{ll})q_l]^+}$, i.e., the left- and right-hand limits of w_{ls} when $y=(w_{ls}+w_{ll})q_l$ are equal. Then we prove that $\frac{dw_{ls}}{dq_l}<0$.

Next, we prove Lemma 5(ii). According to the definitions, we know

$$\begin{cases}\hat{H}(q_l)=\left(q_l\frac{dw_{ls}}{dq_l}+w_{ls}-c_s\right)(1+r_f),\\ \bar{H}(q_l)=p\bar{F}(q_l)\left\{[1-q_l h(q_l)]^2-q_l\left[h(q_l)+q_l h'(q_l)\right]\right\}-(c_s+c_l)(1+r_f).\end{cases}$$

From Equation (17), when $q_l=0$, the maximum $w_{ls}=\frac{p}{1+r_f}-c_l$ and $\hat{H}(q_l=0)=\left(\frac{p}{1+r_f}-c_l-c_s\right)>0$. When $w_{ls}=c_s$, the maximum $q_l=q_\beta$ and $\hat{H}(q_l=q_\beta)=q_\beta\frac{dw_{ls}}{dq_l}(1+r_f)<0$. Similarly, we can easily figure out that $\bar{H}(q_l=0)>0$ and $\bar{H}(q_l=q_\lambda)<0$. Moreover, combined with the proof of Proposition 1, it is obvious that $\frac{d\bar{H}(q_l)}{dq_l}<0$.

When $q_l \in [0, \bar{q}^*]$, $\bar{H}(q_l) = p\bar{F}(q_l)\left\{[1 - q_l h(q_l)]^2 - q_l\,[h(q_l) + q_l h'(q_l)]\right\} - (c_s + c_l)(1 + r_f) \geq 0$,

$$
\begin{aligned}
\hat{H}(q_l) =& \frac{p\bar{F}(q_l)\,[1 - q_l h(q_l)]^2}{1-\delta_l} - \frac{p\bar{F}(q_l)h(q_l)q_l}{1-\delta_l} - \frac{p\bar{F}(q_l)h'(q_l){q_l}^2}{1-\delta_l} - (c_s + c_l)(1 + r_f) \\
&+ \frac{(w_{ls} + c_l)(1 + r_f)h(z_l)\bar{F}(q_l)[1 - q_l h(q_l)]q_l}{\bar{F}(1-\delta_l)^2} \\
&+ \frac{(w_{ls} + c_l)(w_{ls} + w_{ll})(1 + r_l)(1 + r_f){q_l}^2 h'(z_l)\bar{F}(q_l)[1 - q_l h(q_l)]}{p\bar{F}(z_l)(1-\delta_l)^2} \\
=& \frac{\bar{H}(q_l)}{1-\delta_l} + \frac{\delta_l}{1-\delta_l}(c_s + c_l)(1 + r_f) + \frac{(w_{ls} + c_l)(1 + r_f)h(z_l)\bar{F}(q_l)[1 - q_l h(q_l)]q_l}{\bar{F}(1-\delta_l)^2} \\
&+ \frac{(w_{ls} + c_l)(w_{ls} + w_{ll})(1 + r_l)(1 + r_f){q_l}^2 h'(z_l)\bar{F}(q_l)[1 - q_l h(q_l)]}{p\bar{F}(z_l)(1-\delta_l)^2}.
\end{aligned}
$$

Thus, we know $\hat{H}(q_l) > \bar{H}(q_l) \geq 0$ and Lemma 5 is proved.

□

Proof of Proposition 4. The proof of Proposition 4 is similar to that of Proposition 3(ii), so we omit it.

□

Proof of Proposition 5. From Lemma 5, we know $\hat{H}(q_l) > \bar{H}(q_l)$ for $q_l \in [0, \bar{q}^*]$. Thus, it is obvious that $\hat{q}^* > \bar{q}^*$. As a result, $q_l^* \geq \bar{q}^* = q_b^*$.

Next, we prove that $\hat{q}^* < q_c$. From Equation (17), we know

$$
\bar{F}(\hat{q}^*) = \frac{(w_{ls}^* + c_l)\,(1 + r_f)}{p\frac{1-\hat{q}^* h(\hat{q}^*)}{1-\delta_l}}.
$$

From the definition, the order quantity that coordinates the supply chain satisfies $\bar{F}(q_c) = \frac{(c_s+c_l)(1+r_f)}{p}$. Since $p\frac{1-\hat{q}^* h(\hat{q}^*)}{1-\delta_l} < p$ and $w_{ls}^* > c_s$, we have $\bar{F}(\hat{q}^*) > \bar{F}(q_c)$, i.e., $\hat{q}^* < q_c$. Then we conclude that $q_l^* < q_c$ and the supply chain cannot be coordinated.

□

Proof of Proposition 6. We first analyze the supplier's preference between the two financing schemes. Under 3PL financing, we consider the case that $w_{ls} = w_{bs}^*$, where w_{bs}^* is the supplier's optimal wholesale price under bank financing. According to Equations (7) and (17), we can easily figure out that $q_l(w_{ls} = w_{bs}^*) > q_b^*$. Then combined with the supplier's objective function $\Pi_i(w_{is}) = (w_{is} - c_s)q_i(1 + r_f)$, $i = b, l$, we know $\Pi_l(w_{ls} = w_{bs}^*) > \Pi_b(w_{bs}^*)$. However, as w_{ls}^* is the supplier's optimal decision under 3PL financing, we conclude that $\Pi_l(w_{ls}^*) > \Pi_l(w_{ls} = w_{bs}^*) > \Pi_b(w_{bs}^*)$, i.e., the supplier prefers the 3PL financing scheme.

To find out the 3PL's preference, we first need to prove that, given w_{ls}^*, when the 3PL sets $w_{ll} = w_{bl}(q_b^*)^*$ and $r_l = r_b^*(q_b^*)$, the retailer's optimal order quantity $q_l^* > q_b^*$, i.e., the retailer orders more products under 3PL financing.

When the retailer is financed by the bank, the bank determines the bank loan interest rate according to the following equation:

$$[(w_{bs}^* + w_{bl}^*)\, q_b^* - y]\,(1 + r_f) = [(w_{bs}^* + w_{bl}^*) q_b^* - y]\,(1 + r_b^*)\bar{F}(z_b) + \int_0^{z_b} (px + S) f(x) dx, \tag{A12}$$

which is equivalent to

$$1 + r_f = (1 + r_b^*)\bar{F}(z_b) + \frac{\int_0^{z_b} (px + S) f(x) dx}{(w_{bs}^* + w_{bl}^*) q_b^* - y}, \tag{A13}$$

where $z_b = \frac{[(w_{bs}^* + w_{bl}^*) q_b^* - y](1 + r_b^*) - S}{p}$ is the retailer's bankruptcy threshold under bank financing.

Under 3PL financing, the first-order derivative of $\pi_l(q_l)$ with respect to q_l is

$$\frac{d\pi_l(q_l)}{dq_l} = p\bar{F}(q_l) - (w_{ls} + w_{ll})(1 + r_l)\bar{F}(z_l). \tag{A14}$$

Given that $w_{ll} = w_{bl}^*(q_b^*)$ and $r_l = r_b^*(q_b^*)$, combined with Equation (A13) and $p\bar{F}(q_b^*) = (w_{ls}^* + w_{bl}^*)(1 + r_f)$, at point q_b^*, Equation (A14) can be written as

$$\left.\frac{d\pi_l(q_l)}{dq_l}\right|_{q_l = q_b^*} = (w_{ls}^* + w_{bl}^*) \frac{\int_0^{z_b} (px + S) f(x) dx}{(w_{ls}^* + w_{bl}^*) q_b^* - y} > 0.$$

However, in the presence of capital constraints, $\pi_l(q_l)$ is a concave function. Thus the retailer's optimal order quantity under 3PL financing $q_l^* > q_b^*$.

Now, we come back to the 3PL's problem. Under the two financing schemes, the 3PL's objective functions are

$$\begin{cases} \Gamma_b(w_{bl}) = (w_{bl} - c_l) q_b (1 + r_f), \\ \Gamma_l(w_{ll}, r_l) = \min\{p \min(q_l, D) + S, [(w_{ls} + w_{ll}) q_l - y]\,(1 + r_l)\} - [(w_{ls} + c_l) q_l - y](1 + r_f). \end{cases}$$

Given the supplier's wholesale price w_{ls}^*, under 3PL financing, if the 3PL sets $w_{ll} = w_{bl}^*(q_b^*)$, $r_l = r_b^*(q_l^*)$, then

$$\begin{aligned} \Gamma_l(w_{ll} = w_{bl}^*, r_l = r_b^*) &= [(w_{ls}^* + w_{bl}^*) q_l^* - y]\,(1 + r_f) - [(w_{ls}^* + c_l) q_l^* - y]\,(1 + r_f) \\ &= (w_{bl}^* - c_l) q_l^* (1 + r_f) > \Gamma_b(q_b^*). \end{aligned}$$

The equation in the first line holds because of Equation (2). The ">" holds because $w_{ll} = w_{bl}^*$ and $q_l^* > q_b^*$. However, as $(w_{ll}^*, r_l^* = r_f)$ is the 3PL's optimal choice, we conclude that $\Gamma_l(w_{ll}^*, r_l^*) > \Gamma_l(w_{ll} = w_{bl}^*, r_l = r_b^*) > \Gamma_b(w_{bl}^*)$.

Finally, in terms of the retailer's preference between the two financing modes, given the supplier's and 3PL's decisions w_{ls}^* and w_{ll}^*, as the interest rate $r_l^* < r_b^*$, the retailer will choose the 3PL financing scheme.

□

Proof of Proposition 7. We first discuss the 3PL's expected profit. Under 3PL financing and given the supplier's decision w_{ls}^*, if the 3PL adjusts the transportation price and financing interest rate so that $w_{ls}^* + w_{ll} = w_{bs}^* + w_{bl}^*$ and $r_l = r_b^*$ are satisfied, according to our analyses in the proof of Proposition 6, we have $q_l > q_b^*$. Then, as we showed that q_l decreases in $w_{ls} + w_{ll}$ in Lemma 3(i), we can increase w_{ll} to the point w_φ so that $q_l(w_{ls}^*, w_\varphi, r_b^*) = q_b^*$ is satisfied. Consequently, we have

$$z_b = \frac{[(w_{bs}^* + w_{bl}^*)q_b^* - y]\,(1 + r_b^*) - S}{p}, \quad z_l' = \frac{[(w_{ls}^* + w_\varphi)q_l - y]\,(1 + r_b^*) - S}{p} > z_b.$$

The 3PL's expected profit under bank financing and 3PL financing can be rewritten as

$$\begin{cases} \Gamma_b(w_{bl}^*) = \mathbb{E}[p\min(z_b, D)] + S - w_{bs}^* q_b^*(1 + r_f) - (c_l q_b^* - y)(1 + r_f), \\ \Gamma_l(w_{ll} = w_\varphi, r_l = r_b^*) = \mathbb{E}[p\min(z_l', D)] + S - w_{ls}^* q_l(1 + r_f) - (c_l q_l - y)(1 + r_f). \end{cases}$$

We first assume that $w_{ls}^* < w_{bs}^*$. Under this assumption, we know $\Gamma_l(w_{ll} = w_\varphi, r_l = r_b^*) > \Gamma_b(w_{bl}^*)$ is satisfied. Meanwhile, as $\Gamma_l(w_{ll}^*, r_l^*) \geq \Gamma_l(w_{ll} = w_\varphi, r_l = r_b^*)$, it is obvious that the 3PL obtains more expected profit with 3PL financing. Next, we prove that even if $w_{ls}^* \geq w_{bs}^*$, the 3PL is better off under 3PL financing as long as wholesale price w_{ls}^* is not too high. We rewrite the 3PL's expected profit under 3PL financing as

$$\Gamma_l(w_{ll}, r_l) = \int_0^{z_l} pxf(x)dx + \int_{z_l}^{N} pz_l f(x)dx + S - [(w_{ls} + c_l)q_l - y](1 + r_f).$$

Taking the first-order derivative of $\Gamma_l(w_{ll}, r_l)$ with respect to w_{ls}, we get

$$\frac{d\Gamma_l(w_{ll}, r_l)}{dw_{ls}} = p\bar{F}(z_l)\frac{dz_l}{dw_{ls}} - q_l(1 + r_f) - (w_{ls} + c_l)(1 + r_f)\frac{dq_l}{dw_{ls}}.$$

From Equation (13a) and the definition of z_l, we obtain

$$\frac{(w_{ls} + w_{ss})(1 + r_f)f(z_l)}{p}\frac{dz_l}{dw_{ls}} = \frac{(1 + r_f)\bar{F}(z_l)}{p}\left(1 + \frac{dw_{ll}}{dw_{ls}}\right) + f(q_l)\frac{dq_l}{dw_{ls}},$$
$$\frac{dz_l}{dw_{ls}} = \frac{q_l(1 + r_f)}{p}\left(1 + \frac{dw_{ll}}{dw_{ls}}\right) + \frac{(w_{ls} + w_{ll})(1 + r_f)}{p}\frac{dq_l}{dw_{ls}}.$$

Then, we know $\frac{dz_l}{dw_{ls}} = \frac{\bar{F}(q_l)[1 - q_l h(q_l)]}{\bar{F}(z_l)(1 - \delta_l)}\frac{dq_l}{dw_{ls}}$ and

$$\begin{aligned} \frac{d\Gamma_l(w_{ll}, r_l)}{dw_{ls}} &= \left\{\frac{p\bar{F}(q_l)[1 - q_l h(q_l)]}{1 - \delta_l} - (w_{ls} + c_l)(1 + r_f)\right\}\frac{dq_l}{dw_{ls}} - q_l(1 + r_f) \\ &= -q_l(1 + r_f) < 0. \end{aligned}$$

The second "=" holds because of Equation (18). Thus, we know $\Gamma_l(w_{ll}, r_l)$ monotonously decreases in w_{ls}. When $w_{ls}^* \geq w_{bs}^*$, there exists a small non-negative term Θ, such that $\Gamma_l(w_{ll}^*, r_l^*) \geq \Gamma_l(w_\varphi, r_b^*) \geq$

$\Gamma_b(w_{bl}^*)$ holds if $w_{ls}^* \le w_{bs}^* + \Theta$ and Θ satisfies $\Gamma_b(w_{bl}^*) - \Gamma_l(w_\varphi, r_b^* | w_{ls}^* = w_{bs}^* + \Theta) = 0$, i.e.,

$$\mathbb{E}\left\{p \min\left[\frac{(w_{bs}^* + \Theta + w_\varphi)\, q_b^* - y}{p}(1 + r_b^*), D\right]\right\} = \mathbb{E}\left[p \min(z_b, D)\right] + \Theta q_b^*(1 + r_f). \tag{A15}$$

As we mentioned before, when the retailer's unit purchasing cost equals $w_{ls}^* + w_\varphi$ (i.e., $w_{bs}^* + \Theta + w_\varphi$) and $r_l = r_b^*$, $q_l(w_{bs}^* + \Theta, w_\varphi, r_b^*) = q_b^*$ holds. Then, according to the retailer's optimal response functions under the two financing schemes, we know w_φ satisfies

$$(w_{bs}^* + \Theta + w_\varphi)(1 + r_b^*)\bar{F}\left(\frac{\left[(w_{bs}^* + \Theta + w_\varphi)q_b^* - y\right](1 + r_b^*) - S}{p}\right) = (w_{bs}^* + w_{bl}^*)(1 + r_f). \tag{A16}$$

By solving the Equations (A15) and (A16), we can get the value of Θ.

Finally, in terms of the retailer's problem, based on Equations (3) and (12a), we can rewrite its expected profits under the two financing schemes as

$$\begin{cases} \pi_b(q_b^*) = \mathbb{E}\left\{p \min(q_b^*, D) - \left[(w_{bs}^* + w_{bl}^*)\, q_b^* - y\right](1 + r_b^*) + S\right\}^+ - y(1 + r_f) - S, & \text{(A17a)} \\ \pi_l(q_l^*) = \mathbb{E}\left\{p \min(q_l^*, D) - \left[(w_{ls}^* + w_{ll}^*)\, q_l^* - y\right](1 + r_f) + S\right\}^+ - y(1 + r_f) - S. & \text{(A17b)} \end{cases}$$

From Equations (4), (13a), (A17a), and (A17b), if $(w_{ls}^* + w_{ll}^*) < (w_{bs}^* + w_{bl}^*)$, it is obvious that $q_l^* > q_b^*$ and $\pi_l(q_l^*) > \pi_b(q_b^*)$. From Lemma 3(i) we know $\pi_l(q_l^*)$ monotonously decreases in $(w_{ls}^* + w_{ll}^*)$. When $(w_{ls}^* + w_{ll}^*) \ge (w_{bs}^* + w_{bl}^*)$, there exists a small non-negative term Δ, such that $\pi_l(q_l^*) > \pi_b(q_b^*)$ holds if $(w_{ls}^* + w_{ll}^*) < (w_{bs}^* + w_{bl}^*) + \Delta$ and Δ satisfies $\mathbb{E}\left\{p \min(q_\phi, D) - \left[(w_{bs}^* + w_{bl}^* + \Delta)\, q_\phi - y\right](1 + r_f)\right\}^+ = \mathbb{E}\left[p \min(q_b^*, D)\right] - \left[(w_{bs}^* + w_{bl}^*)\, q_b^* - y\right](1 + r_f)$, where q_ϕ solves

$$\bar{F}(q_\phi) = \frac{(w_{bs}^* + w_{bl}^* + \Delta)(1 + r_f)}{p}\bar{F}\left(\frac{\left[(w_{bs}^* + w_{bl}^* + \Delta)\, q_\phi - y\right](1 + r_f) - S}{p}\right).$$

□

Proof of Proposition 8. We first analyze the case that $\underline{\mathbb{C}} \le y \le \bar{\mathbb{C}}$. In this case, both $\bar{q}^*$ and $\hat{q}^*$ can be the optimal order quantity. From the definition of $\underline{\mathbb{C}}$, we know $\hat{q}^* \ge \bar{q}^*$. If $q_l^* = \bar{q}^*$, then $q_l^* = q_b^*$, $w_{ls}^* + w_{ll}^* = w_{bs}^* + w_{bl}^*$, and $\pi_l(q_l^*) = \pi_b(q_b^*)$. If $q_l^* = \hat{q}^*$, then $q_l^* \ge q_b^*$. Combined with Lemma 3(i) and 3(iii), we have $w_{ls}^* + w_{ll}^* \le w_{bs}^* + w_{bl}^*$ and $\pi_l(q_l^*(w_{ls}^*, w_{ll}^*)|r_l^*) \ge \pi_l(q_l^*(w_{bs}^* + w_{bl}^*)|r_l^*) \ge \pi_l(q_l^*(w_{bs}^* + w_{bl}^*)|r_b^*) = \pi_b(q_l^*(w_{bs}^* + w_{bl}^*)|r_b^*)$.

To prove our conclusion under the case of $\underline{\underline{\mathbb{C}}} \le y < \underline{\mathbb{C}}$, we need to study the case when $y \to 0$. From the definition of z_l, we have $z_l = \frac{[(w_{ls}^* + w_{ll}^*)\hat{q}^* - y](1 + r_l^*) - S}{p} = \frac{(w_{ls}^* + w_{ll}^*)\hat{q}^*(1 + r_l^*)}{p} - \frac{y(1 + r_l^*) + S}{p}$. From the retailer's optimal response function in Equation (13a), $\hat{q}^*\bar{F}(\hat{q}^*) - z_l\bar{F}(z_l) - \frac{y(1 + r_l^*) + S}{p}\bar{F}(z_l) = 0$. Recall that $qh(q)$ is an increasing function for $q \in [0, q_\alpha]$ and $z_l < \hat{q}_l^* < q_\alpha$. As a result, we have $z_l \to \hat{q}^*$ as $y \to 0$. The retailer's expected profit in Equation (12a) can be rewritten as $\pi_l(\hat{q}^*) = \mathbb{E}\left[p \min(\hat{q}^*, D) - pz_l\right]^+ - y(1 + r_l^*) - S$. Then we know $\pi_l(\hat{q}^*) \to 0 < \pi_b(q_b^*)$. Meanwhile, since $y \to 0$ and $z_l \to \hat{q}^*$, from Equation (13a) we know $\frac{(w_{ls}^* + w_{ll}^*)(1 + r_l^*)}{p} \to 1$. That is, $(w_{ls}^* + w_{ll}^*) \to \frac{p}{1 + r_l^*} \ge \frac{p}{1 + r_b^*} \ge$

$(w_{bs}^* + w_{bl}^*)$. Since $(w_{ls}^* + w_{ll}^*)$ is continuous in y, there exists a $0 < \underline{\underline{\mathbb{C}}} < \underline{\mathbb{C}}$, such that $w_{ls}^* + w_{ll}^* \leq w_{bs}^* + w_{bl}^*$ and $\pi_l(q_l^*(w_{ls}^*, w_{ll}^*)|r_l^*) \geq \pi_l(q_l^*(w_{bs}^* + w_{bl}^*)|r_l^*) \geq \pi_l(q_l^*(w_{bs}^* + w_{bl}^*)|r_b^*) = \pi_b(q_l^*(w_{bs}^* + w_{bl}^*)|r_b^*)$ for $\underline{\underline{\mathbb{C}}} \leq y < \underline{\mathbb{C}}$.

□

Proof of Proposition 9. We expand Equation (22) and obtain

$$\begin{aligned}
\Gamma_l(w_{ll}, r_l) =& \mathbb{E}\left[\min\left\{p\min(q_l, D) + S, [(w_{ls} + w_{ll})q_l - y]^+(1 + r_l)\right\}\right] \\
&+ \mathbb{E}\left\{[Y + \min(y, (w_{ls} + w_{ll})q_l) - (w_{ls} + c_l)q_l]^+(1 + r_f)\right\} \\
&- \mathbb{E}\left\{[(w_{ls} + c_l)q_l - Y - \min(y, (w_{ls} + w_{ll})q_l)]^+(1 + r_f)\right\} - Y(1 + r_f) \\
=& \mathbb{E}\left[\min\left\{p\min(q_l, D) + S, [(w_{ls} + w_{ll})q_l - y]^+(1 + r_l)\right\}\right] \\
&+ \mathbb{E}\left[\min(y, (w_{ls} + w_{ll})q_l)(1 + r_f)\right] - (w_{ls} + c_l)q_l(1 + r_f). \qquad \text{(A18)}
\end{aligned}$$

Meanwhile, from Equation (15), we can rewrite the 3PL's expected profit when it has sufficient capital as

$$\begin{aligned}
\Gamma_l(w_{ll}, r_l) =& \mathbb{E}\left[\min\left\{p\min(q_l, D) + S, [(w_{ls} + w_{ll})q_l - y]^+(1 + r_l)\right\}\right] \\
&+ \mathbb{E}\left[\min(y, (w_{ls} + w_{ll})q_l)(1 + r_f)\right] - (w_{ls} + c_l)q_l(1 + r_f). \qquad \text{(A19)}
\end{aligned}$$

It is obvious that Equations (A18) and (A19) are the same, which means the 3PL's expected profit is independent of Y. Consequently, we know the 3PL's decisions on transportation price and 3PL financing interest rate are not affected by its initial capital level.

□

Proof of Proposition 10. We omit the proof of Proposition 10(i) because it is similar to the proof of Proposition 2.

For Proposition 10(ii), the supplier's objective function is $\tilde{\Pi}_l(\tilde{w}_{ls}) = (\tilde{w}_{ls} - c_s)\tilde{q}_l(1 + r_f)$. From the retailer's response function in Proposition 10(i), we know there is a one-to-one mapping between $\tilde{w}_{ls}$ and $\tilde{q}_l$. Then $\tilde{w}_{ls}$ can be seen as a function of $\tilde{q}_l$ and $\frac{d\tilde{w}_{ls}}{d\tilde{q}_l} = \frac{p\bar{F}(\tilde{q}_l)[\tilde{\delta}_l - \tilde{q}_l h(\tilde{q}_l)]}{q\bar{F}(\tilde{z}_l)(1+\tilde{r}_l)(1-\tilde{\delta}_l)}$, where $\tilde{\delta}_l = (\tilde{w}_{ls} + \tilde{w}_{ll})(1 + \tilde{r}_l)p\tilde{q}_l h(\tilde{z}_l)/p$. By taking the first-order derivative of $\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))$ with respect to $\tilde{q}_l$ and substituting $d\tilde{w}_{ls}/d\tilde{q}_l$, we have

$$\frac{d\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))}{d\tilde{q}_l} = \frac{p\bar{F}(\tilde{q}_l)(1 + r_f)}{\bar{F}(\tilde{z}_l)(1 + \tilde{r}_l)} \frac{1 - \tilde{q}_l h(\tilde{q}_l)}{1 - \tilde{\delta}_l} - (\tilde{w}_{ll} + c_s)(1 + r_f).$$

Then we analyze the second-order derivative of $\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))$ by taking the first-order derivative of

$\ln\left(\frac{\bar{F}(\tilde{q}_l)(1-\tilde{q}_l h(\tilde{q}_l))}{\bar{F}(\tilde{z}_l)(1-\tilde{\delta}_l)}\right)$ with respect to $\tilde{q}_l$, and obtain

$$\begin{aligned}
&\frac{d}{d\tilde{q}_l}\ln\left(\frac{\bar{F}(\tilde{q}_l)(1-\tilde{q}_l h(\tilde{q}_l))}{\bar{F}(\tilde{z}_l)(1-\tilde{\delta}_l)}\right)\\
=&-h(\tilde{q}_l)+h(\tilde{z}_l)\frac{1-\tilde{q}_l h(\tilde{q}_l)}{1-\tilde{\delta}_l}\frac{\bar{F}(\tilde{q}_l)}{\bar{F}(\tilde{z}_l)}-\frac{h(\tilde{q}_l)}{1-\tilde{q}_l h(\tilde{q}_l)}+\frac{h(\tilde{z}_l)}{1-\tilde{\delta}_l}\frac{1-\tilde{q}_l h(\tilde{q}_l)}{1-\tilde{\delta}_l}\frac{\bar{F}(\tilde{q}_l)}{\bar{F}(\tilde{z}_l)}\\
&-\frac{\tilde{q}_l h'(\tilde{q}_l)}{1-\tilde{q}_l h(\tilde{q}_l)}+\frac{\tilde{q}_l h'(\tilde{z}_l)}{1-\tilde{\delta}_l}\frac{1-\tilde{q}_l h(\tilde{q}_l)}{1-\tilde{\delta}_l}\left[\frac{\bar{F}(\tilde{q}_l)}{\bar{F}(\tilde{z}_l)}\right]^2.
\end{aligned}$$

Since $\tilde{q}_l > \tilde{z}_l$, $h(\tilde{q}_l) > h(\tilde{z}_l)$, $\tilde{q}_l h(\tilde{q}_l) > \tilde{\delta}_l$, and $h'(\tilde{q}_l) > h'(\tilde{z}_l)$, we know that $\frac{d}{d\tilde{q}_l}\ln\left(\frac{\bar{F}(\tilde{q}_l)(1-\tilde{q}_l h(\tilde{q}_l))}{\bar{F}(\tilde{z}_l)(1-\tilde{\delta}_l)}\right) < 0$. That means $\frac{d\tilde{\Pi}_l^2(\tilde{w}_{ls}(\tilde{q}_l))}{d\tilde{q}_l^2} < 0$ and $\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))$ is concave. Meanwhile, as $\left.\frac{d\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))}{d\tilde{q}_l}\right|_{\tilde{q}_l=0} > 0$ and $\left.\frac{d\tilde{\Pi}_l(\tilde{w}_{ls}(\tilde{q}_l))}{d\tilde{q}_l}\right|_{\tilde{q}_l=q_\alpha} < 0$, we conclude that the optimal $\tilde{q}_l$ to the supplier satisfies the first-order condition, i.e.,

$$\frac{p\bar{F}(\tilde{q}_l)}{\bar{F}(\tilde{z}_l)}\frac{1-\tilde{q}_l h(\tilde{q}_l)}{1-\tilde{\delta}_l}-(\tilde{w}_{ll}+c_s)(1+\tilde{r}_l)=0.$$

□